The due date for the assignment is 6 March.

1. (3 points) Using Fermat’s Little theorem, find the residue of $2^{1,000,000}$ modulo 7.

2. (3 points) Find the last digit of the base 7 expansion of $3^{100}$.

3. (4 points) Let $p$ be a prime and $k$ be an integer such that $0 < k < p$. Show that $(p - k)!(k - 1)! \equiv (-1)^k \pmod{p}$.

4. (3 points) Show that every composite number $F_m = 2^{2^n} + 1$ is a pseudoprime to the base 2.

5. (3 points) Let $m$ be a positive integer. Find a reduced residue system modulo $2^m$.

6. (4 points) Show that 1387 is a pseudoprime but not a strong pseudoprime to the base 2.