The due date for the assignment is 27 January.

1. (5 points) Which of the following sets are well-ordered? Justify your answer in each case.
   (a) The set of natural numbers, \( \mathbb{N} = \{1, 2, 3, \cdots\} \).
   (b) The open interval \((-1, 1)\).
   (c) The set of positive rational numbers.
   (d) The set of integers, \( \mathbb{Z} \).
   (e) A proper subset of a well-ordered set.

2. (4 points) Let \( r = -\frac{30}{4} \). Write down the answer to each of the following:
   (a) The floor function of \( r \), \( \lfloor r \rfloor \).
   (b) The ceiling function of \( r \), \( \lceil r \rceil \).
   (c) The integer part of \( r \), \( \lfloor r \rfloor \).
   (d) The fractional part of \( r \), \( \{ r \} \).

3. (3 points) Which of the following sequences can be defined by a recursive relation? If yes, write down a recursive relation defining that sequence.
   (a) The sequence of Fibonacci numbers, \((0, 1, 1, 2, 3, 5, 8, \cdots)\).
   (b) The sequence \((0, \frac{1}{2}, 1, \frac{3}{2}, 4, \cdots)\) defined by \( \{a_n\}_{n \geq 0} \) where \( a_n = \left(\frac{n}{2}\right)^2 \) if \( n \) is even and \( a_n = \frac{n}{2} \) if \( n \) is odd.
   (c) The sequence of squares \((0, 1, 4, 9, \cdots)\) defined by \( \{a_n\}_{n \geq 0} \), \( a_n = n^2 \).

4. (4 points) Let \( r = \sqrt{5} \). Find integers \( a \) and \( b \) such that \( |ar - b| < \frac{1}{3} \).

5. (4 points) Show that 3 divides \( n^3 - n \) for any integer \( n \). Does 4 divide \( n^4 - n \) for any integer \( n \)? Prove this or provide a counterexample.

6. (5 points) Use induction to prove that \( n^2 \leq 2^n \) for all natural numbers \( n \geq 4 \).