Math 302 Midterm 2

Instructor: Prof. Ed Perkins Duration: 50 minutes.

Instructions:

- Write your name and student ID on **every** page.
- This examination contains three questions on five pages worth a total of 101 points.
- Write each answer **very clearly** below the corresponding question (Use back of page if needed). Simplify your answer as much as possible.
- No calculators, books, notebooks or any other written materials are allowed.
- **Good luck!**

1. (a) (16 points) A random variable X has cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{2} & \text{if } 0 \leq x < \frac{1}{4} \\ \frac{3}{2}x & \text{if } \frac{1}{4} \leq x < \frac{1}{2} \\ 1 & \text{if } x \geq \frac{1}{2}. \end{cases}$$

(i) Find $P(X = \frac{1}{2})$.

(ii) Find $P(X < \frac{1}{4})$.

(b) (17 points) Assume Y has cumulative distribution function

$$F_Y(y) = \begin{cases} 1 - y^{-4} & \text{if } y \geq 1 \\ 0 & \text{if } y < 1. \end{cases}$$

(i) Find the probability density function of Y.

(ii) Find the variance of Y.

2. (34 points) Let R be the rotated square in the $x-y$ plane with corners at $(-1, 0)$, $(0, -1)$, $(1, 0)$ and $(0, 1)$. Assume (X, Y) is uniformly distributed over R, that is, X and Y have a joint density which is a constant c on R, and equal to 0 on the complement of R.

   (a) Find c.

   (b) Find the marginal densities of X and Y.

   (c) Are X and Y independent? Justify your answer.

   (d) Find $P(Y > |X|)$.

3. (34 points) Assume Z_1, Z_2, X_1, and X_2 are independent random variables, where Z_1 and Z_2 are standard normal,while X_1 and X_2 are exponential r.v.'s with rate $\lambda = 1/2$. Find the probability density function of the following random variables.

   (a) e^{X_1}

   (b) $2Z_1 + 3Z_2$

   (c) $Z_1^2 + Z_2^2 + X_1 + X_2$.

(d) $X_1 + Z_1$. Your answer may left in terms of the function $\Phi(b) = \int_{-\infty}^{b} \frac{e^{-z^2/2}}{\sqrt{2\pi}}\, dz$.