

Math 302 Midterm 1

Instructor: Prof. Ed Perkins

Duration: 50 minutes.

Instructions:

- Write your name and student ID on **every** page.
- This examination contains four questions with total weight of 101 points.
- Write each answer **very clearly** below the corresponding question (Use back of page if needed). Simplify your answer as much as possible (but answers may be in terms of the exponential function, factorials, or “choose” symbols).
- No calculators, books, notebooks or any other written materials are allowed.
- **Good luck!**

1. (a) (10 pts) Define carefully: A probability P on a given sample space S .

(b) (9 pts) Find the probability that a five card poker hand contains exactly 3 Aces.

(c) (10 pts) A and B are events such that $P(A) = .6$, $P(B) = .3$ and $P(A \cup B) = .8$.
Find $P(A \cap B^c)$.

2. There are 12 buses in the 100 Mile House bus fleet each with a capacity of 30 people. Currently 6 of the buses are running full, 3 of them are running half full and 3 of them are running a sixth full.

(a) (6 pts) If a bus is chosen at random what is the probability that the bus is full?

(b) (8 pts) If a bus rider is chosen at random what is the probability they are on a full bus?

(c) (10 pts) If a bus rider is chosen at random and Y is the number of people on the rider's bus, find $E(Y)$. (You may leave your answer as a sum of explicit fractions.)

3. (a) (15 pts) The mean no. of misprints on a page of area 400 cm^2 is 10. A silver dollar of area 20 cm^2 is tossed on the page at random. Find the approximate probability that it covers at least one misprint.

- (b) (15 pts) A fair die is tossed. If the outcome is n then n fair dice are tossed and Y is the sum of these n dice. If we are given that $Y = 3$ find the probability that Y is the sum of 2 dice.

4. X_1 is a binomial r.v. with parameters $n = 6$, $p = \frac{1}{3}$; X_2 is a geometric r.v. with parameter $p = \frac{1}{4}$, and X_3 is a Poisson r.v with parameter $\lambda = 3$. Assume X_1, X_2 and X_3 are independent r.v.'s.

(a) (8 pts) Find $\text{Var}(X_1 + X_2 + X_3 + 1)$.

b) (10 pts) Show that $P(X_2 > X_1) = \left(\frac{11}{12}\right)^6$.