

Math 302 practice midterm 2

Instructor: Prof. Ed Perkins

Duration: 50 minutes.

Instructions:

- Write your name and student ID on **every** page.
- This examination contains four questions worth a total of 100 points.
- Write each answer **very clearly** below the corresponding question (Use back of page if needed). Simplify your answer as much as possible (but answers may be in terms of elementary functions such as the exponential function).
- No calculators, books, notebooks or any other written materials are allowed.
- **Good luck!**

2. (a) (7 pts) Define carefully: The cumulative distribution function of a r.v. X .

(b) A r. v. X has cumulative distribution function

$$F(x) = \begin{cases} 1 - e^{-x^2/2} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

(i) (7 pts) Find $P(1 \leq X \leq 2)$.

(ii) (10 pts) Find $E(X)$.

(c) (10 pts) If X and Y are independent random variables with cumulative distribution functions F_X and F_Y , respectively, what is the cumulative distribution function of $\min(X, Y)$, where $\min(X, Y)$ is the minimum of X and Y .

3. Give examples of the following:

(a) (10 pts) A continuous r.v. which has no mean value. (Give the probability density function explicitly.)

(b) (10 pts) A pair of continuous r.v.'s X and Y , which do not have a joint probability density function. Justify your answer.

4. Let X, Y be random variables with joint density function given by

$$f(x, y) = \begin{cases} c & \text{if } x^2 + y^2 \leq 16, y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) (10 pts) What is the value of c ?

(b) (16 pts) Let (θ, R) be (X, Y) in polar coordinates (that is, $(X, Y) = (R \cos \theta, R \sin \theta)$, where $R \geq 0$, $\theta \in [0, \pi]$). Compute the joint cumulative distribution function of θ and R , and the joint probability density function of θ and R . Are θ and R independent?