

Math 302 practice midterm 1

Instructor: Prof. Ed Perkins

Duration: 50 minutes.

Instructions:

- Write your name and student ID on **every** page.
- This examination contains four questions with weight 25 points each.
- Write each answer **very clearly** below the corresponding question (Use back of page if needed). Simplify your answer as much as possible (but answers may include factorials, or “choose” symbols).
- No calculators, books, notebooks or any other written materials are allowed.
- **Good luck!**

1. Lois sends her resume to 1000 companies she found at monster.com. Each company responds with probability $3/1000$ (independently of what all other companies do). Let R be the number of companies that respond.

(a) Compute $E(R)$ (give an exact answer, not an approximation).

(b) Compute $\text{Var}(R)$ (give an exact answer, not an approximation).

(c) Use a Poisson random variable approximation to estimate the probability $P(R = 3)$.

2. (a) Consider a game in which you toss a fair die and win \$3 for each time your roll a 5 or 6 but lose \$1 for each time you roll 4 or less. You get to play this game 6 times for an entry fee. What would be a fair entry fee?

- (b) How many ways are there to divide 15 books into five groups of size 1, 2, 3, 4 and 5.

- (c) How many five card poker hands are there with exactly four ranks. (Recall there are 13 ranks and four cards of each rank in standard deck of 52 cards.)

3. It is known that A and B are events such that $P(A) = 0.6$ and $P(A \cup B) = 0.8$. Find $P(B)$ in each of the following cases:

(a) A and B are disjoint.

(b) A and B are independent.

(c) $P(B|A) = .3$.

4. (a) What is the probability mass function for a geometric random variable with parameter p . State your answer carefully.

- (b) An infinite number of independent trials are conducted, each have probability p to succeed. Let $r \geq 1$ be an integer and X be the r.v. counting the number of trials conducted until r successes have been seen. Hint: write $X = Y_1 + \dots + Y_r$.

(i) Calculate $E[X]$. Justify your answer.

(ii) Calculate $\text{Var}(X)$. Justify your answer.