Solutions to Practice Questions (in Ch6 only)

P 289 #6.28 (a) X_1 = service time for A.J.

X_2 = service time for M.J.

X_1, X_2 independent exponential (1) so joint density is

$$f(x_1,x_2) = e^{-x_1-x_2} 1_{\{x_1>0,\, x_2>0\}}.$$

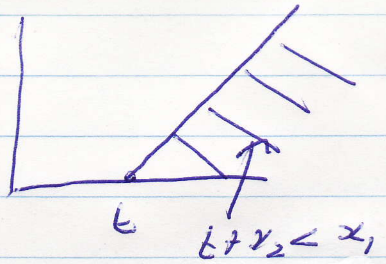
P(M.J.'s car done before A.J.'s)

$= P(t + X_2 < X_1)$

$$= \iint_{\{t+x_2<x_1\}} 1_{\{x_1,x_2\geq 0\}} e^{-x_1-x_2}\, dx_1\, dx_2$$

$$= \int_0^\infty \left[\int_{t+x_2}^\infty e^{-x_1} dx_1\right] e^{-x_2} dx_2$$

$$= \int_0^\infty e^{-t-x_2} e^{-x_2} dx_2 = \underline{\underline{\frac{e^{-t}}{2}}}$$



(b) $X_1 + X_2$ is gamma(2,1) $\therefore$ has density $x e^{-x} 1_{\{x>0\}}$

$$P(X_1+X_2<2) = \int_0^2 \underset{f}{x}\, \underset{dg}{\underline{e^{-x}dx}} \qquad g = -e^{-x} \quad f' = 1$$

$$= -x e^{-x}\Big|_0^2 + \int_0^2 e^{-x} dx$$

$$= -2e^{-2} + 1 - e^{-2} = \underline{1-3e^{-2}}.$$

#6.29 (a) $X_1 + X_2$ is $N(4400,\ 230^2+230^2) = N(4400,\ 2(230)^2)$

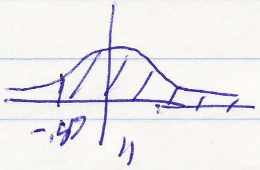
$$\text{So } P(X_1+X_2 > 5000) = P\left(\frac{X_1+X_2-4400}{\sqrt{2}\cdot 230} > \frac{5000-4400}{\sqrt{2}\cdot 230}\right)$$

$$= P(Z > 1.845) \qquad Z\ N(0,1)$$

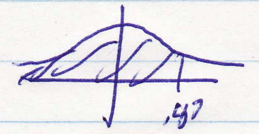
$$= 1 - \Phi(1.845) \approx 1 - .9674 \approx .0326.$$

(X_1 = sales in week 1, X_2 = sales in week 2.)

(b) $P(X_1 > 2000) = P\left(\frac{X_1 - 2200}{230} > \frac{-200}{230}\right)$



$= P(Z \geq -.87)$ $\quad Z \sim N(0,1)$



$= P(Z \leq .87)$

$= \Phi(.87) \approx .8078$

N = # weeks in next 3 sales exceed 2000

N is binomial $(3, .8078 = p)$

$\therefore P(N \geq 2) = \binom{3}{2}(.8078)^2(1-.8078) + \binom{3}{3}(.8078)^2$

$= .376 + .527$

$= \underline{.903}$