Math 302 Assignment 9

This assignment is not to be handed but you should do all the questions in preparation for the final exam.

1. If $X_1, X_2, \ldots, X_n$ is a random sample from an unknown distribution with (unknown) mean μ and variance σ^2. Suppose we are willing to assume that $\sigma^2 \leq 10$. How large should n be to ensure that the sample mean $\overline{X}_n$ is within .1 of the mean μ with probability at least .95? Use the Central Limit Theorem this time and compare your answer with that of Q 6 in Assignment 8 where you used Chebychev's Inequality.

2. Twenty-five heat lamps are connected in a greenhouse so that when one lamp fails another takes over immediately (only one lamp is turned on at any time). The lamps operate independently and each has a mean lifetime of 50 hours and a standard deviation of 4 hours. If the greenhouse is not checked for 1300 hours after the lamp system is turned on, what is the approximate probability that a lamp will be burning at the end of the 1300-hour period?

3. Assume X has variance 1, Z is a standard normal r.v. independent of X, and $Y_\epsilon = a + bX + \epsilon Z$, where $b \neq 0$. Show that $\rho(X, Y_\epsilon) = \frac{b}{\sqrt{b^2+\epsilon^2}}$. Find $\lim_{\epsilon\to\infty} \rho(X, Y_\epsilon)$ and $\lim_{\epsilon\to 0} \rho(X, Y_\epsilon)$. (This may illustrate how $\rho(X, Y)$ can "measure" the degree of linear dependence between X and Y.)

4. Assume $X_1, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ r.v.'s. Show that $\hat{X}_n = \frac{\bar{X}_n - \mu}{(\sigma/\sqrt{n})}$ is a standard normal r.v.

Conclude that the standard normal distribution is the only possible limiting distribution in the Central Limit Theorem. That is if someone tells you the Central Limit is true for some limiting distribution, then the limiting distribution must be standard normal.

5. A gambler pays \$40 and then tosses a fair die and wins \$10 times the outcome of the die. Approximiate P(gambler is winning after playing this game 30 times).

6. Consider 20 randomly chosen people.
(a) Find P(at least one of the people has a birthday on Jan. 1). Ignore leap years.
(b) If N is the number of different birthdays among the 20 people find $E(N)$. Ignore leap years.
Hint. Let $X_j = 1$(day j is a birthday of at least one of the 20 people) for $j = 1, \ldots, 365$.

Here are some practice problems to try before the exam.
p. 226 #5.26, 5.28, 5.29
p. 413 #8.6, 8.15
p. 415 #8.1, 8.2, 8.3(a), 8.5, 8.8