Math 302 Assignment 8

This assignment is due in class on Wednesday November 23.

1. Assume X_1 and X_2 are independent exponential r.v.'s with rates λ_1 and λ_2, respectively. If $\lambda_1 > \lambda_2$, find the probability density function of $X_1 + X_2$.

2. Let $X_1, X_2, \ldots, X_{10}$ be uncorrelated r.v.'s, each with mean value 0 and variance 2.
(a) Find an upper bound for $P(|X_j| \geq 2)$ (for all j).
(b) Find an upper bound for $P(|X_1 + X_2 + \ldots + X_{10}| \geq 20)$.
(c) If, in addition, you know that $E((X_j)^4) \leq 6$ for all j, can you improve the upper bound in (a)? (If so, give the improvement.)

3. Let X be the number of even tosses and Y the number of 3's that occur in n rolls of a fair die. Compute $\text{Cov}(X, Y)$.

4. The times between radioactive decays (α particle emissions) of a substance are independent exponential r.v.'s with rate λ. That is if T_n be the time between the $n-1$st decay and nth decay, where T_1 is the time of the first decay, then T_1, T_2,... are independent rate λ exponential r.v.'s.
(a) Find the probability density function of the time S_n until the nth decay.
(b) Find $\text{Cov}(S_m, S_n)$ where $m \leq n$.
(c) Find the joint probability density function of S_n and T_{n+1}.
(d) Use (b) to find $P(\text{exactly } n \text{ decays before time } u)$. What is the distribution of the discrete r.v. $N(u) =$ number of decays up to time u?

5. Customers arrive in single server queue to be serviced. That is, there is a single server who services each customer in the order they arrive while all the customers wait in line. The times between customer arrivals are are independent exponential r.v.'s with mean 5. The service times of the customers are independent uniform r.v.'s on $[0, 4]$. The service times are also independent of the inter-arrival times.

Find the expected value of the time that the second customer will have to wait before being served.

6. If $X_1, X_2, \ldots, X_n$ is a random sample from an unknown distribution with (unknown) mean μ and variance σ^2. Suppose we are willing to assume that $\sigma^2 \leq 10$. How large should n be to ensure that the sample mean $\overline{X}_n$ is within .1 of the mean μ with probability at least .95?

Here are some practice problems not to be handed in, but try them before the exam.

p. 412 # 8.1, 8.2
p.373 # 7.5, 7.37, 7.38, 7.39

A dartboard centered at the origin has radius 6. Let (X, Y) be the random location of a dart thrown by a competent player. Assume X and Y have joint probability density function

$$f(x, y) = \begin{cases} c(6 - \sqrt{x^2 + y^2}), & \text{if } \sqrt{x^2 + y^2} \leq 6 \\ 0, & \text{if } \sqrt{x^2 + y^2} > 6 \end{cases}.$$

This is to help you brush up on your polar coordinates.
(a) Find c.
(b) The bullseye is the center circle of radius $1/2$. Find the probability that the player hits the bullseye.