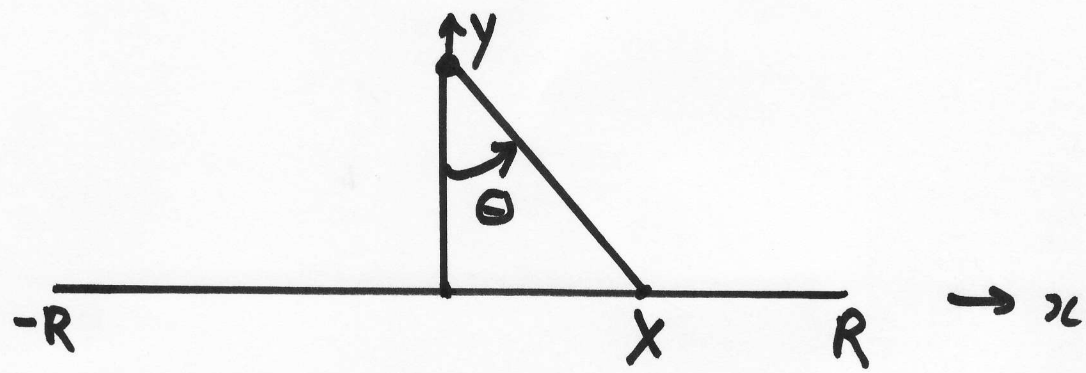
Math 302 Assignment 7

This assignment is due in class on Wednesday November 9. Late assignments can only be handed in during the office hour on Thursday November 10 2:15-3:45.

1. If X is a Cauchy r.v. find the p.d.f of $\sqrt{|X|}$.

2.



Consider the wall (viewed from above in the diagram) stretching from $-R$ to R. It is illuminated by a search light, unit distance from the wall, sweeping with constant angular velocity from one end of the wall to other. Let θ be the angle the searchlight makes with the y-axis and let X be the of the position the centre of the spotlight on the wall, as shown. At any instant in time we may assume that θ has a uniform distribution on its range (which is no longer $[-\pi/2, \pi/2]$).
(a) Find the pdf of X.
(b) Show that $P(|X| \leq 1) \geq \frac{1}{2}$ no matter how long the wall is.
(c) Find the mean and variance of X.

3. The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} cx, & \text{if } 0 < y \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find c.
(b) Find $P(X \leq \frac{1}{2}, Y \geq \frac{1}{4})$.
(c) Find $P(|Y - X| \leq \frac{1}{2})$.
(d) Find the marginal pdf's of X and Y. Are X and Y independent? Justify your answer.

4. The joint probability density function of X and Y is given by

$$f(x,y) = x + y \qquad 0 \leq x \leq 1,\ 0 \leq y \leq 1\,.$$

(f is 0 elsewhere.)
(a) Find the marginal p.d.f.'s of X and Y.
(b) Are X and Y independent? Justify your answer
(c) Find the cumulative distribution function of $X + Y$.

5. Customers arrive in single server queue to be serviced. That is, there is a single server who services each customer in the order they arrive while the other customers wait in line.

The times between customer arrivals are are independent exponential r.v.'s with mean 5. The service times of the customers are independent uniform r.v.'s on $[0, 4]$. The service times are also independent of the inter-arrival times.

Find the probability that the second customer to arrive will have to wait before being served.

Here are some practice problems not to be handed in, but try them before the second midterm.

p. 227 # 5.38, 5.39, 5.2
p. 294-295 # 6.3, 6.6, 6.12
p. 287-288 # 6.10, 6.13, 6.14, 6.16, 6.19, 6.20, 6.27 (here find the cdf of Z and $P(X_1 < X_2)$)

Assume that F is a strictly increasing and continuous function satisfying

$$\lim_{x \to \infty} F(x) = 1, \qquad \lim_{x \to -\infty} F(x) = 0.$$

In particular, F satisfies the conditions (i)-(iii) given in class which we claimed characterized cumulative distribution functions. The point of this question is to exhibit a r.v. with c.d.f. F in this special case.

Let F^{-1} be the inverse function of F and let U be a uniform r.v. on $(0, 1)$. Show that $X = F^{-1}(U)$ is a r.v. with cumulative distribution function F.

In general if F satisfies only the conditions (i)-(iii) (increasing, normalization, and right-continuity) and for $y \in (0, 1)$ we define $F^{-1}(x)$ to be the least upper bound of

$$\{x \in \mathbf{R} : F(x) < y\},$$

then the above conclusion still holds. But you don't have to show this.