

Math 302 Assignment 5

This assignment is due on Wed. October 26.

1. In each of the following cases, compute $P(0 < X < 2)$ where the random variable X has the given probability density function.

(a)

$$f(x) = \begin{cases} x^{-2}, & \text{if } x \geq 1, \\ 0, & \text{if } x < 1. \end{cases}$$

(b)

$$f(x) = \begin{cases} 7e^{-7x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

2. Suppose that the random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{8}x & \text{if } 0 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the value of a such that $P(X \leq a) = \frac{1}{2}$.

(b) Determine the value of a such that $P(X \geq a) = \frac{1}{4}$.

3. Assume X has probability density function

$$f(x) = \begin{cases} cx^4 & \text{if } 0 < x < a \\ 0 & \text{otherwise,} \end{cases}$$

for some positive constants c and a . If $E(X) = 1$, find $Var(X)$.

4. An electronic system consists of 5 components which operate independently. The lifetime (in years) of each component is an exponential r.v. with parameter $\lambda = \frac{1}{2}$.

(a) If N is the number of components still operating after 4 years, find the expectation of N .

(b) The system will work as long as at least two of the components are still functioning. Find $P(\text{system will still be working in 4 years})$.

5. (a) For each of the following r.v.'s X calculate the function $F(x) = P(X \leq x)$ and then graph $y = F(x)$.

(i) X uniform on $(0, 10)$.

(ii) X exponential, $\lambda = 3$.

(iii) X Binomial ($n = 5, p = .5$).

(b) In each of the above cases, verify by direct calculation that

$$E(X) = \int_0^{\infty} P(X > x) dx.$$

Here are some practice problems not to be handed in, but try them before the second midterm.

p. 229-230 # 5.1, 5.2, 5.7

p. 224-225 #5.5, 5.7, 5.10, 5.14 (just use Proposition 2.1 (LUS)).