Math 302 Assignment 5
This assignment is due on Wed. October 26.

1. In each of the following cases, compute $P(0 < X < 2)$ where the random variable $X$ has the given probability density function.
   (a) 
   \[ f(x) = \begin{cases} 
   x^{-2}, & \text{if } x \geq 1, \\
   0, & \text{if } x < 1.
   \end{cases} \]
   (b) 
   \[ f(x) = \begin{cases} 
   7e^{-7x}, & \text{if } x \geq 0, \\
   0, & \text{if } x < 0.
   \end{cases} \]

2. Suppose that the random variable $X$ has probability density function
   \[ f(x) = \begin{cases} 
   \frac{1}{8}x, & \text{if } 0 \leq x \leq 4, \\
   0, & \text{otherwise}.
   \end{cases} \]
   (a) Determine the value of $a$ such that $P(X \leq a) = \frac{1}{2}$.
   (b) Determine the value of $a$ such that $P(X \geq a) = \frac{1}{4}$.

3. Assume $X$ has probability density function
   \[ f(x) = \begin{cases} 
   cx^4, & \text{if } 0 < x < a \\
   0, & \text{otherwise},
   \end{cases} \]
   for some positive constants $c$ and $a$. If $E(X) = 1$, find $Var(X)$.

4. An electronic system consists of 5 components which operate independently. The lifetime (in years) of each component is an exponential r.v. with parameter $\lambda = \frac{1}{2}$.
   (a) If $N$ is the number of components still operating after 4 years, find the expectation of $N$.
   (b) The system will work as long as at least two of the components are still functioning. Find $P($system will still be working in 4 years$)$.

5. (a) For each of the following r.v.'s $X$ calculate the function $F(x) = P(X \leq x)$ and then graph $y = F(x)$.
   (i) $X$ uniform on $(0, 10)$.
   (ii) $X$ exponential, $\lambda = 3$.
   (iii) $X$ Binomial $(n = 5, p = .5)$.
   (b) In each of the above cases, verify by direct calculation that
   \[ E(X) = \int_0^\infty P(X > x) \, dx. \]

Here are some practice problems not to be handed in, but try them before the second midterm.

p. 229-230 # 5.1, 5.2, 5.7
p. 224-225 #5.5, 5.7, 5.10, 5.14 (just use Proposition 2.1 (LUS)).