

Math 302 Assignment 3

This assignment is due in class on Wed. October 5.

1. Coin 1 has $P(H) = \frac{1}{3}$ and Coin 2 has $P(H) = \frac{3}{4}$. A coin is picked at random and then flipped 5 times.

(a) Find the probability of getting exactly 3 Heads.

(b) If you get exactly 3 Heads, find the probability that you were flipping Coin 2.

2. Due to noise interference a communication channel transmits a bit (0 or 1) correctly with probability $\frac{4}{5}$. Each bit is transmitted independently. To reduce the error the sender transmits 00000 or 11111, and the receiver will record 0 if a majority of 0's are received and 1 if a majority of 1's are received. Find the probability that a bit is transmitted correctly with this new protocol.

3. Assume X and Y are independent random variables each taking on the values 1 and -1 with probability $\frac{1}{2}$. Let $Z = XY$. Show that X, Y and Z are not independent random variables but are pairwise independent (that is, each pair of them are independent).

4. It is known that $P(A_1) = 1/10$, $P(A_2) = 1/50$, and $P(A_3) = 1/100$.

(a) If the above three events are independent, find $P(A_1 \cup A_2 \cup A_3)$.

(b) If we only know the above three events are pairwise independent, show that

$$.1268 \leq P(A_1 \cup A_2 \cup A_3) \leq .127.$$

5. Prove that $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$ for all natural numbers n .

6. A fair coin is tossed n times. Show carefully that

$$P(\text{there is an odd number of Heads}) = \frac{1}{2}.$$

Here are some practice problems not to be handed in, but try them before the first midterm.

p. 172-176 # 4.1, 4.4, 4.6, 4.8 (a,b), 4.18, 4.40, 4.44, 4.46

If 1_A is the indicator function of an event A , prove that two events A and B are independent if and only if their indicator functions are independent random variables. This extends easily to finitely many events.

Show that 1_A is a Bernoulli r.v. with $p = P(A)$. (This is a one line application of the definition!)