Points

Math 302 Assignment 2 Solutions

1. $T = \{\text{tourist}\}$ $C = \{\text{correct answer}\}$

$P(T) = 2/3$ $P(C|T) = 3/4$ $P(C|T^c) = 1/10$

(2) (a) $P(C) = P(C|T)P(T) + P(C|T^c)P(T^c) = \frac{3}{4}\cdot\frac{2}{3} + \frac{1}{10}\cdot\frac{1}{3} = \frac{16}{30} = \frac{8}{15}$.

(2) (b) $P(T^c|C) = \dfrac{P(C|T^c)P(T^c)}{P(C)} = \dfrac{\frac{1}{10}\cdot\frac{1}{3}}{8/15} = \frac{1}{30} \times \frac{15}{8} = \frac{1}{16}$.

2. (a) $B_j = \{\text{select blue ball on } j^{th} \text{ draw}\}$

$R_j = B_j^c = \{\text{select red ball on } j^{th} \text{ draw}\}$

(2) $P(1^{st}\text{ red on }3^{rd}\text{ draw}) = P(B_1 \cap B_2 \cap R_3)$

$= P(B_1)P(B_2|B_1)P(R_3|B_1 \cap B_2)$

$= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$

(b) ~~Methods~~ Distinguish between blue balls: $b_1, \ldots, b_6$ and red balls: $r_1, \ldots, r_4$.

(2)

Method 1. S = all ordered arrangements of 10 distinct balls

Each ordered draw is equally likely.

$\therefore P(A) = \dfrac{\#A}{\#S} = \dfrac{\#A}{10!}$ Let $A = \{\text{last ball red}\}$

$B = \{\text{first ball red}\}$.

There is a 1-1 map between A and B -- just reverse the order.

$\therefore \#A = \#B$

$\therefore P(A) = \dfrac{\#A}{\#S} = \dfrac{\#B}{S} = P(1^{st}\text{ ball red}) = \frac{4}{10} = \frac{2}{5}$.

or

<u>Method 2.</u> last ball red iff 1^{st} 9 balls consist of 6 blue and 3 red.

So Experiment: Draw 1^{st} 9 balls.

S = all 9 ball subsets of 10 balls

Each subset equally likely.

$A = \{$ choose 6 blue and 3 red $\}$

$$P(A) = \frac{\# A}{\# S} = \frac{\binom{6}{6}\binom{4}{3}}{\binom{10}{9}} \qquad \text{(as for poker hands)}$$

$$= \frac{4}{10} = \frac{2}{5}.$$

(c) S = all 4 ball subsets of our 10 balls

A = set of all all 4 ball subsets with 2 b_i's and 2 r_i's

2 $$P(A) = \frac{\# A}{\# S} = \frac{\binom{4}{2}\binom{6}{2}}{\binom{10}{4}} = \frac{3}{7}.$$

3. (a) <u>False</u>

Take $\Omega = \{H, T\}$ and $P(\{H\}) = P(\{T\}) = \frac{1}{2}$

2 $\{H\} \cap \{T\} = \emptyset$ so they are disjoint.

But $P(\{H\} \cap \{T\}) = P(\emptyset) = 0 \neq \frac{1}{4} = P(\{H\})P(\{T\})$

$\therefore \{H\}, \{T\}$ are dependent.

(b) True. Let A be uninteresting, B any event.

<u>Case 1.</u> $P(A) = 0$.

Then $P(A \cap B) \leq P(A) = 0$

2 $\therefore P(A \cap B) = 0 = P(A)P(B) \Rightarrow A, B$ independent.

Case 2. $P(A) = 1$

$P(A^c) = 1 - P(A) = 0 \Rightarrow A^c$ and B independent by Case 1

$\therefore (A^c)^c$ and B are independent (shown in class)

$\therefore A$ and B are independent.

(c) True.

Since $A \subset B$, we have $A \cap B = A$ and so $P(A \cap B) = P(A)$

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$\therefore P(A \cap B) - P(A)P(B) = P(A) - P(A)P(B) = P(A)[1 - P(B)]$

since $P(A) > 0$ and $P(B) < 1$.

4. (a) You and your spouse each have a type a gene because your daughter has blue eyes and each have a type A gene because you each have brown eyes.
Hence you each have a type aA gene pair.

3 So the experiment is randomly selecting 1 type from you and from your spouse. $S = \{aa, aA, AA\}$.

Let X be the pair type of your son.

$P(X = aa) = P(\text{choose } a \text{ from each parent}) = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$

$P(X = AA) = P(\quad " \quad A \quad) = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$

$P(X = aA) = P(\text{choose } a \text{ from mother, } A \text{ from father}) + P(\text{choose } A \text{ from mother, } a \text{ from father})$

$= \frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2} = \frac{1}{2}$.

So $P(\{\text{Brown eyes}\}) = P(\{AA, aA\}) = \frac{1}{4} + \frac{1}{2} = 3/4$.

(b) $P(\text{blue-eyed grandchild} \mid \text{son has brown eyes})$

$$= \frac{P(\{\text{blue-eyed grandchild}\} \cap \{X = aA \text{ or } AA\})}{P(\text{son has brown-eyes})}$$

$$= \frac{\left[P(\text{blue-eyed grandchild} \mid X = aA)\, P(X = aA) + P(\text{blue-eyed grandchild} \mid X = AA)\, P(X = AA)\right]}{3/4}$$

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(from (a))

$$= \left[\left(\tfrac{1}{2} \times \tfrac{1}{2}\right) + \left(0 \times \tfrac{1}{4}\right)\right] / \tfrac{3}{4}$$

$$= 1/3$$

Here we used the fact that your son's wife has blue eyes so that $P(\text{blue-eyed grandchild} \mid X = aA) = \frac{1}{2}$.

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