1. (a) \( S = 1, 2, \ldots, 365^3 \).

We record the birthdays of the 3 people in order, and ignore Feb. 29.

(b) \( A = \text{all 3 have same birthday} \).

We assume each outcome in \( S \) is equally likely.

\[
\begin{align*}
P(A) &= \frac{\#A}{\#S} = \frac{365 \times 1 \times 1}{365^3} = \frac{1}{365^2} = \frac{1}{133,225}. \\
\end{align*}
\]

2. Suppose you choose \( 1, 3, 3, 4, 4, 5, 5, \) say.

\( A = \text{get exactly 3 balls correct} \).

\[
\begin{align*}
\#A &= \text{ways to choose 3 balls from 51, 63, } \\
&= \binom{51}{3} \times \binom{63}{3}.
\end{align*}
\]

\[
\begin{align*}
P(A) &= \frac{\binom{51}{3} \times \binom{63}{3}}{\binom{6}{3}} \\
&= \frac{51 \times 43 \times 42 \times 61 \times 60 \times 59}{3! \times 3! \times 40 \times 31 \times 49!} \\
&\approx 0.1765.
\end{align*}
\]

3. Roll \( n \) dice. \( S = 1, \ldots, 6^n \).

\( A = \text{roll at least one 6} \).

\( A^c = \text{roll no 6's} \).

\[
P(A) = 1 - P(A^c) = 1 - \frac{5^n}{6^n}.
\]

\[
\begin{align*}
P(A) > \frac{1}{2} \iff P(A^c) < \frac{1}{2} \iff \left(\frac{5}{6}\right)^n < \frac{1}{2} \iff \left(\frac{6}{5}\right)^n > 2 \\
\iff n \log \frac{6}{5} > \log 2 \iff n > \frac{\log 2}{\log \left(\frac{6}{5}\right)} \approx 3.80.
\end{align*}
\]

So you need roll 4 dice.
Method 1. Label 5 A's as \( A_1, A_2, \ldots, A_5 \)
2 B's as \( B_1, B_2 \)
2 R's as \( R_1, R_2 \)

Now all 11 letters are distinct.

S = all ordered arrangements of 11 (distinct) letters
\[ \# S = 11! \]

\( E \) = arrangement spells \( A_2 R_3 A_1 A_5 C A_4 D A_3 B R_2 A \)

\( \# B = 4 \) (always to put 5 A's in 5 slots for A's)
\( \# R = 2 \) (always to put 2 B's in 2 slots for B's)
\( \# A = 2 \) (always to put 2 R's in 2 slots for R's)

\[ = 5! \times 2! \times 2! \]

\[ P(E) = \frac{5! \times 2! \times 2!}{11!} \approx 1.20 \times 10^{-5} \]

Method 2. \( S' = \) all distinct spellings of arrangements of \( A_2 R_3 A_1 A_5 C A_4 D A_3 B R_2 A \)

\[ \# S' = \frac{11!}{5! \times 2! \times 2!} \]
(see Eq. 3d, p. 4)

By symmetry each spelling will be equally likely, so

\[ P(\{ A_2 R_3 A_1 A_5 C A_4 D A_3 B R_2 A \}) = \frac{1}{\# S'} = \frac{5! \times 2! \times 2!}{11!} \approx 1.20 \times 10^{-5} \]
\[ F = [(A, UA_2) \cap A_3] \cup A_4 \]

\[ = [(A \cap A_3) \cup (A_2 \cap A_3) \cup A_4] \]  
(\text{Distributive property of } \cup \text{ over } \cap) 

So by inclusion-exclusion

\[ P(F) = P(A, \cap A_3) + P(A_2 \cap A_3) + P(A_4) 
- P(A, \cap A_3 \cap A_4) - P(A_2 \cap A_3 \cap A_4) - P(A, \cap A_3 \cap A_2 \cap A_3) 
+ P(A, \cap A_3 \cap A_2 \cap A_4) \]

\[ = \frac{1}{4} + \frac{1}{4} + \frac{1}{2} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} + \frac{3}{16} = \frac{11}{16}. \]

ib) Now \[ S = 3H, T \bar{T} \bar{S}^2, \]
\[ (A, UA_2) \cap A_3 = \frac{1}{4} H H, H T \bar{T} S \bar{T} S H \bar{S} = \emptyset \]

So \[ F = A_4 \quad \therefore P(F) = P(A_4) = P(S^2 T T S) = \frac{1}{4}. \]