Solutions to HW 302 - #1

1. (a) $S = \{1, 2, .., 365\}^3$.

We record the birthdays of the 3 people in order, and ignore Feb. 29.

1b) $A = \{$all 3 have same birthday$\}$

We assume each outcome in S is equally likely.

$$\therefore P(A) = \frac{\# A}{\# S} = \frac{365 \times 1 \times 1}{365^3} = \frac{1}{365^2} = \underline{\frac{1}{133{,}225}}.$$

2. Suppose you choose $\{1, 3, 3, 4, 5, 6\}$, say.

$A = \{$get exactly 3 balls correct$\}$

$\# A = \#$ ways to choose 3 balls from $\{1, .., 6\} \times \#$ ways to choose 3 from $\{7, .., 49\}$.

$$= \binom{6}{3} \times \binom{43}{3}$$

So $$P(A) = \frac{\binom{6}{3} \times \binom{43}{3}}{\binom{49}{6}} = \frac{6!\ 43!\ 43!\ 6!}{3!\ 3!\ 40!\ 3!\ 49!} \approx \underline{.01765}.$$

3. Roll n dice. $S = \{1, .., 6\}^n$.

$A = \{$roll at least one 6$\}$. $A^c = \{$roll no 6's$\}$.

$P(A^c) = \frac{\# A^c}{6^n} = \frac{5^n}{6^n}$. $P(A) = 1 - P(A^c)$

So $P(A) > \frac{1}{2} \Leftrightarrow P(A^c) < 1/2 \Leftrightarrow \left(\frac{5}{6}\right)^n < \frac{1}{2} \Leftrightarrow \left(\frac{6}{5}\right)^n > 2$

$\Leftrightarrow n \log \frac{6}{5} > \log 2 \Leftrightarrow n > \frac{\log(2)}{\log(6/5)} \doteq 3.80$

So you need to roll 4 dice.

4. <u>Method 1.</u> Label 5A's as $A_1, \ldots, A_5$
2 B's as B_1, B_2
2 R's as R_1, R_2.

Now all 11 letters are distinct.

S = all ordered arrangements of 11 (distinct) letters

$\# S = 11!$

E = {arrangement spells ABRACADABRA}

$\# E$ = #(ways to put 5 A_i's in 5 slots for A's)
× #(ways to put 2 B_i's in 2 slots for B's)
× #(ways to put 2 R_i's in 2 slots for R's)
$= 5! \times 2! \times 2!$

$$P(E) = \frac{5!\,2!\,2!}{11!} \approx 1.20 \times 10^{-5}.$$

<u>Method 2.</u> S' = all distinct spellings of rearrangements of ABRACADABRA

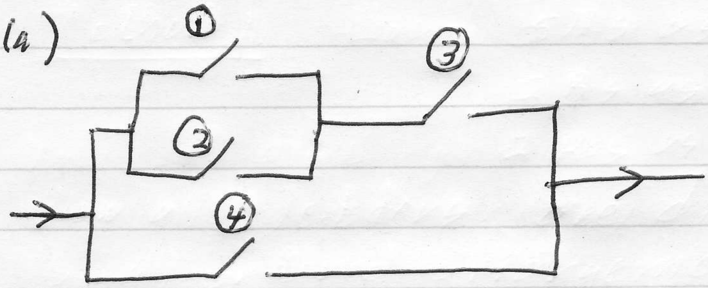
$$\# S' = \frac{11!}{5!\,2!\,2!}$$

(see Eg 3d, p. 4).

By symmetry each spelling will be equally likely, so

$$P(\{ABRACADABRA\}) = \frac{1}{\# S'} = \frac{5!\,2!\,2!}{11!} \approx 1.20 \times 10^{-5}$$

5 (a)



$A_i = \{\text{switch } i \text{ is on}\}$
$= \{i^{th} \text{ toss is } H\}$.
$F =$ event that current flows.

$$F = [(A_1 \cup A_2) \cap A_3] \cup A_4$$

$$= [(A_1 \cap A_3) \cup (A_2 \cap A_3) \cup A_4]$$ (Distributive property of $\cup$ over $\cap$)

So by inclusion exclusion

$$P(F) = P(A_1 \cap A_3) + P(A_2 \cap A_3) + P(A_4)$$
$$- P(A_1 \cap A_3 \cap A_4) - P(A_2 \cap A_3 \cap A_4) - P(A_1 \cap A_3 \cap A_2 \cap A_3)$$
$$+ P(A_1 \cap A_3 \cap A_2 \cap A_4)$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{2} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} + \frac{1}{16} = \frac{11}{16}.$$

(b) Now $S = \{H, T\}^2$.

$$(A_1 \cup A_2) \cap A_3 = \{HH, HT\} \cap \{TH\} = \emptyset$$

So $F = A_4$ $\therefore P(F) = P(A_4) = P(\{TT\}) = \frac{1}{4}$.