Math 302 Assignment 10

This assignment is not to be handed but you should do all the questions in preparation for the final exam.

1. An opinion poll takes a random sample of 1350 voters in a large city. Suppose that 40% of the population support Party A, while 0.2% of the population support Party B. Find numerical approximations (using different distributions) for
   (a) the probability that more than 544 of the people in the sample support Party A, and
   (b) the probability that at most 2 of the people in the sample support Party B.

2. Let $X_n$ be a binomial $(n, p_n)$ r.v. where $np_n \to \lambda > 0$ as $n \to \infty$.
   (a) Find the m.g.f. $M_{X_n}(t)$ of $X_n$.
   (b) Show directly from (a) that for each $t$, \( \lim_{n \to \infty} M_{X_n}(t) = M_X(t) \), where $X$ is a Poisson r.v. with mean $\lambda$.

3. Let $X_1, \ldots, X_n$ be a random sample from a geometric distribution with parameter $p$.
   (a) Find the m.g.f of the sample mean $\bar{X}_n$.
   (b) What should be the limiting value of $M_{\bar{X}_n}(t)$ as $n$ gets large?
   (c) [Bonus question if this was a real assignment.] Show the above.

4. If $X_1, X_2, X_3$ are independent r.v.s with $X_1$ standard normal, $X_2$ exponential with rate 2 and $X_3$ exponential with rate 3 find $E(e^{X_1+X_2+X_3})$.

5. If 12 married couples are randomly seated at 4 different tables (so that 6 people are each table), find the expected number of married couples that are seated at the same table. (See problem 7.3(a) below.)

Here are some practice problems to try before the exam.

p. 379 #7.75
p. 384 #7.3 (a), 7.5