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Theorem (Cauchy-Schwarz) For any r.vs X, Y

$$|E(X \cdot Y)| \le \sqrt{E(X^2)}\sqrt{E(Y^2)}$$

Proof. Let $f(x) = E((xX+Y)^2)$ $x \in \mathbb{R}$.

Then $f(x) = E(x^2X^2 + 2xXY + Y^2)$

$$= \underbrace{E(X^2)}_{a}x^2 + \underbrace{2E(XY)}_{b}x + \underbrace{E(Y^2)}_{c}.$$

By its definition $ax^2+bx+c = f(x) \ge 0$ for all x.

$\therefore$ The discriminant $b^2 - 4ac \le 0$.

ie $b^2 \le 4ac$

$$4E(XY)^2 \le 4E(X^2)E(Y^2)$$

$$\therefore |E(XY)| \le \sqrt{E(X^2)}\sqrt{E(Y^2)} \qquad \square$$