Theorem 4.24. Let $M \in \mathfrak{gl}(M)$. Let $t \geq 0$. Then $M$ is linear from $L_1^2(M)$ to $\mathfrak{gl}(M)$ and
\begin{equation}
H \to H^* \text{ is linear from } L_1^2(M) \text{ to } \mathfrak{gl}(M) \text{ and } \mathfrak{m}.
\end{equation}

(i) If $H = \frac{e}{2} \left( \mathcal{L}(U_1, V_{ij}) \right)$, then
\begin{equation}
H \cdot M = \frac{e}{2} \left( M(U_1, V_{ij}) - M(U_{ij}, V) \right).
\end{equation}

Proof. Let $t \geq 0$. Let $\bar{t} = \frac{1}{2} \left( \mathcal{L}(U_1, V_{ij}) \right)$, and $\bar{H} = \mathcal{L}(U_{ij}, \bar{V})$ where $H, \bar{H} \in L_1^2(M)$. Let $M_\nu = \frac{1}{2} \mathcal{L}(U_1, \nu) + \mathcal{L}(H, \nu) + \mathcal{L}(\bar{H}, \nu)$. By definition:
\begin{equation}
\left( H \cdot M \right)^{\nu} = \left( M^{\nu} \right)^{\nu} = \left( M^{\nu} \right)^{\nu} = H \cdot \left( M^{\nu} \right)^{\nu} = \left( H \cdot M \right)^{\nu} = \left( H \cdot \mathcal{L}(U_1, \nu) + \mathcal{L}(H, \nu) + \mathcal{L}(\bar{H}, \nu) \right)^{\nu} = \left( H \cdot M \right)^{\nu}.
\end{equation}

Similarly, \( (H + \mathcal{K})^{\nu} = \left( H + \mathcal{K} \right)^{\nu} \).
\begin{equation}
(H + \mathcal{K})^{\nu} = \left( H + \mathcal{K} \right)^{\nu} = \left( H + \mathcal{K} \right)^{\nu}.
\end{equation}

Let $n \to 0$. $(H + \mathcal{K})^{\nu} = H^{\nu} + \mathcal{K}^{\nu}$.

Considered. Let $3 \mathcal{L}(U_1, \nu) = H$.
\begin{equation}
(H \cdot M)^{\nu} = H \cdot \left( M^{\nu} \right)^{\nu} = \frac{e}{2} \left( M(U_1, V_{ij}) - M(U_{ij}, V) \right).
\end{equation}

Now let $n \to 0$ to derive (i).