Math 420/507 Assignment 9-not to hand in

1. p. 92 # 10, #13

2. Assume $\nu$ and $\mu$ are measures on $(X, \mathcal{A})$ such that $\mu$ is $\sigma$-finite and $\nu \ll \mu$.
   (a) Prove that for any non-negative measurable function $f$ on $X$, 
   \[
   \int f \, d\nu = \int f \frac{d\nu}{d\mu} \, d\mu.
   \]
   (b) If $\frac{d\nu}{d\mu} > 0$ $\mu$-a.e. prove that $\mu \ll \nu$ and $\frac{d\mu}{d\nu} = \left(\frac{d\nu}{d\mu}\right)^{-1}$. Of course this last statement can only mean they agree $\mu$-a.e.

3. p. 93 #17 (I hope to discuss this question in class.)

4. p. 93 #11 (Here you can use Thm. 3.5 in the text. The $L^1$ “metric” is 
   \[
   \|f - g\|_1 = \int |f - g| \, d\mu.
   \]