1. p. 52 #14

2. Let \( f : \mathbb{R} \to [0, \infty] \) be Lebesgue measurable and \( m \) denote Lebesgue measure. Prove that if \( a \in \mathbb{R} \), then \( \int f(x) \, dm = \int f(x + a) \, dm \).

**Hint:** First prove it for \( f \) simple.

3. p. 49 # 10 Here you should prove that completeness implies (a) and (b), and that (a) alone implies completeness. Also recall the definition of “\( \mu \)-a.e.” on p. 26.

4. Assume \((X, \mathcal{M}, \mu)\) is a measure space and \( f, g : X \to \overline{\mathbb{R}} \) are \( \mathcal{M} \)-measurable. If \( \int f \, d\mu \) and \( \int g \, d\mu \) exist and \( f \leq g \) \( \mu \)-a.e., then show that \( \int f \, d\mu \leq \int g \, d\mu \).

Although the text deals with complex-valued functions you should assume the functions in the questions below are \( \mathbb{R} \)-valued. Unless another measure space is specified you may assume the questions refer to a fixed measure space \((X, \mathcal{M}, \mu)\).

5. p. 59 # 20, 21

6. Let \( f(x) = x^{-1/2}1_{(0,1)}(x) \) for \( x \in \mathbb{R} \) and let \( \{r_n : n \in \mathbb{N}\} \) be an enumeration of the rationals. Define \( g(x) = \sum_{n=1}^\infty 2^{-n}f(x - r_n) \in [0, \infty] \).

   (a) Prove that \( g \in L^+ \) and is integrable w.r.t. Lebesgue measure, \( m \). So in particular \( g < \infty \) a.e.

   (b) Prove that \( g^2 < \infty \) a.e. but is not Lebesgue integrable on any interval of positive length, i.e., the integral \( \int_a^b g^2 \, dm = \infty \) for all \( a < b \).

In (b) you may assume Thm. 2.28–namely that the Lebesgue integral extends the Riemann integral as we will prove in class.