1. Let $X$ be a countable set and let $\mathcal{P}_F(X)$ denote the finite subsets of $X$. Show that $\mathcal{P}_F(X)$ is countable.

2. Let $X$ be a non-empty set and let $\mathcal{R}$ be the set of partial orders on $X$. Show that $\mathcal{R}$ is partially ordered by inclusion ($\subset$) (this is as trivial as it seems). Now show that any total order on $X$ is a maximal element in $(\mathcal{R}, \subset)$.

3. Prove that if every countable subset of a totally ordered set $X$ is well-ordered then $X$ itself is well-ordered.

4. Let $R$ be a partial order on a non-empty set $X$. Prove there is a total order $S$ on $X$ such that $R \subset S$, i.e., $S$ extends $R$.

**Hint:** Prove the converse of the final statement in Q. 2.