1. \[ \int |x_1| \mathbb{I}_{\{|x_1| \geq n\}} dP = \int \frac{|x_1|}{\Phi(|x_1|)} \frac{\Phi(|x_2|)}{\Phi(|x_1|)} \mathbb{I}_{\{x_1 \geq n\}} dP \leq \frac{1}{kn} \sup \| \theta(kn) \| . \]

2. \[ \sup \int |x_1| \mathbb{I}_{\{|x_1| \geq n\}} dP \to 0 \text{ as } n \to \infty. \]

where we used Thm. 5.3 (in lectures) to get the 2nd convergence.

3. \[ S_n = \frac{1}{n} \sum_{i=1}^{n} Y_i \] is a martingale s.t. \( |S_{n+1} - S_n| \leq \alpha. \)

So by Cor. 5.4 (in lectures) if
\[ C = \{ \lim_{n \to \infty} S_n \text{ exists in } \mathbb{R} \}, \quad D = \{ \lim_{n \to \infty} S_n = -\infty \text{ and } \lim_{n \to \infty} S_n = \infty \}, \]

then \( (x) \in \mathbb{P}(C \cup D) = 1, \)

Note that \( C = \{ m \in \mathbb{R}, \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} Y_i \text{ exists in } \mathbb{R} \} \subseteq \mathbb{D}(Y_1, Y_2, \ldots) \). \( \forall \alpha \in \mathbb{R}. \)

\[ C \subseteq T = \{ \delta(Y_1, Y_2, \ldots) \}. \]

By Kolmogorov 0-1 law \( \mathbb{P}(C) = 0 \text{ or } 1. \)

Case 1: \( \mathbb{P}(C) = 1 \) so (a) holds

Case 2: \( \mathbb{P}(C) = 0 = \mathbb{P}(D) = 1 \) (by (b)), so (b) holds.
4. Fix a version of $\Pi(1/n)$ for all $n \in \mathbb{N}$.

By hypothesis, $\exists n_0, \forall n \geq n_0 \Pi(n) = 0$ and $\forall n_0$.

(i) $\Pi(1/n)$ is decreasing.

(ii) $\lim_{n \to \infty} \Pi(1/n)$ exists.

(iii) $\lim_{n \to \infty} \Pi(1/n) = 1$, $\forall n \in \mathbb{N}$.

(The last (since $0 < 1$). Let $\omega = \lim \Pi(1/n).$

(iv) $\Pi(1/n) \leq \lim_{n \to \infty} \Pi(1/n) = 1$.

5. Assume $\lim_{n \to \infty} x_n$ is ergodic. Let $A \in \mathcal{A}$.

Since $x_n = \frac{dQ}{dP}(\cdot)$, $Q(A) = \int_A x_n dP = \int_A \frac{dQ}{dP} x_n dP = \int_A x_n dP = \lim_{n \to \infty} \int_A x_n dP$.

Hence $Q(A) = \lim_{n \to \infty} x_n dP = \lim_{n \to \infty} \frac{dQ}{dP} x_n = 0$. Similarly, $\forall n$.

$m = \{A \in \mathcal{A} | Q_A = 1\}$ holds $0 = 0$ and is a monotone class.

By the monotone class theorem $Q_A$ holds $\forall A \in \mathcal{A}$, $Q_A = 0$.

Recall (2) is $Q|_{\mathcal{A}} \ll P|_{\mathcal{A}}$, $\forall \mathcal{A}$.