Proposition 1.31 (b) \( \text{If } \{x_n:n \in \mathbb{N}\} \text{ are e.v.s on } (\Omega, \mathcal{F}, P) \), then \( C = \limsup_{n \to \infty} x_n \) exists in } \mathbb{R}^{\infty} \\

Proof. Recall \( d(x, y) = \|x - y\| \) \( \forall x, y \in \mathbb{R} \).
\[
C = \{ \lim x_n : \liminf x_n \in \mathbb{R}^{\infty} \}
\]

(1) \(-\frac{1}{2} \leq \{\liminf x_n, \lim x_n\} \leq \frac{1}{2}\)

\((x, y) \to d(x, y)\) is convex on \(\mathbb{R} \times \mathbb{R}\) and \(\liminf, \lim x_n\) are e.v.'s. (last line), so just use \( d(x, y) w = d(\liminf x_n, \lim x_n) \) is a n.v. (Only diff for 1r. Use \( x = \lim x_n, \lim x_n\) is a \(\mathbb{R}^2\)-valued r. vector, as in Cont. 29, we use \( \mathbb{R}^2 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \).

This shows the 1st set in (b) is in it. I claim that \( 0 = \{0, 0, 0, 0\} \) and \( \lim x_n \) is a.e. e.v. shows the 2nd set in (b) is also in it. So \( C \in \mathbb{F} \) by (1). \( \square \)