MATHEMATICS 320, FALL 2016, PROBLEM SET 3
Due on Friday, October 14, in class

Write clearly and legibly, in complete sentences. You must provide complete explanations for all your solutions; answers without justification, even if correct, will not be marked. You may discuss the homework with other students, but the final write-up must be your own.

1. Prove that $(0, 1) \sim \mathbb{R}$.

2. Recall the field $\mathbb{Q}(\sqrt{3}) = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$ from the last assignment. Prove that $\mathbb{Q}(\sqrt{3})$ and $\mathbb{Q}$ have the same cardinality.

3. (a) Prove that any infinite set contains a countable set.
   (b) If $A$ is an infinite set and $B$ is at most countable, prove that $A$ and $A \cup B$ have the same cardinality.

4. Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers. Suppose that $\{a_n\}$ has a subsequence convergent to $a$, and $\{b_n\}$ has a subsequence convergent to $b$. Must the sequence $\{a_n + b_n\}$ have a subsequence convergent to $a + b$? Prove your answer.

5. Evaluate the following using the definition of limit:
   (a) $\lim_{n \to \infty} \frac{n}{(n + 1)^{3/2}}$
   (b) $\lim_{n \to \infty} (n^2 + 1)^{1/8} - n^{1/4}$
   (c) $\lim_{n \to \infty} (a^n + b^n)^{1/n}$, where $a, b > 0$.

6. The following questions from the textbook should be done but are NOT to be handed in: Chapter 2 #2, 3, 4; Chapter 3 #3