Write clearly and legibly, in complete sentences. You must provide complete explanations for all your solutions; answers without justification, even if correct, will not be marked. You may discuss the homework with other students, but the final write-up must be your own.

1. (a) Let $F = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$. Prove that $F$ is a field.
   (b) Let $F' = \{a + b\sqrt{2} + c\sqrt{3} : a, b, c \in \mathbb{Q}\}$. Prove that $F'$ is not a field.

2. Find the supremum and infimum of each of the following sets of real numbers, or prove that they do not exist.
   (a) $A = \left\{ \frac{m}{m + n} : m, n \in \mathbb{N} \right\}$
   (b) $B = \left\{ \frac{mn}{1 + m + 2n} : m, n \in \mathbb{N} \right\}$

3. Prove that for any two numbers $x, y \in \mathbb{R}$ such that $0 < x < y$, there are positive integers $m, n$ such that
   \[ x < \frac{m^2}{n^2} < y. \]

4. Prove that for any $\alpha > 0$, and for any complex numbers $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$, we have
   \[ \left| \sum_{j=1}^{n} a_j b_j \right| \leq \frac{1}{\alpha} \sum_{j=1}^{n} |a_j|^2 + \frac{\alpha}{4} \sum_{j=1}^{n} |b_j|^2. \]

5. The following questions from the textbook should be done but are NOT to be handed in: Chapter 1 # 6, 13, 17