Math 302 Practice Midterm 2

Instructor: Ed Perkins

Instructions:

- Write your name and student ID on every page.

- This examination contains four questions worth a total of 50 points.

- Write each answer very clearly below the corresponding question (Use back of page if needed).

- No calculators, books, notebooks or any other written materials are allowed.

- Good luck!
1. (10pts) Evaluate $\int_C (\sin x + y^2)\,dx + (x - e^{-y})\,dy$ where $C$ is the boundary of the semi-circular region, $x^2 + y^2 \leq 4$, $y \geq 0$ with the positive orientation.
2. (10 pts) If $\mathbf{F}$ is a $C^1$ vector field on $\mathbb{R}^3$ and $\phi$ is a $C^1$ scalar field on $\mathbb{R}^3$, show that

\[ \nabla \times (\phi \mathbf{F}) = \phi (\nabla \times \mathbf{F}) + (\nabla \phi) \times \mathbf{F}. \]
3. (20 pts) Let $F(x, y, z) = (ax + \frac{2xy^2}{x^2 + 1} + e^{2z})i + 2y \ln(x^2 + 1)j + bxe^{2z}k$.

(a) Find all values of $a$ and $b$ for which $F$ is a conservative and find a potential for $F$ for each of these values.

(b) Let $C$ be the curve of intersection of the surfaces $y = x$ and $z = y^2$ from $(0, 0, 0)$ to $(1,1,1)$ and oriented in this direction. If

$$G(x, y, z) = \left(x + \frac{2xy^2}{x^2 + 1} + e^{2z}\right)i + (2y \ln(x^2 + 1) + xy)j + 2xe^{2z}k,$$

find $\int_C G \cdot dr$. 
4. (10 pts) A $C^1$ simple, closed, positively oriented curve $C$ in the plane has centroid $(6, 1)$. If $\int_C 3y^2 \, dx + (x^2 + x) \, dy = 3$, find the area of the $I(C)$, that is the area of the inside of $C$. 