

# Math 302 practice midterm 1

Instructor: Ed Perkins

Duration: 50 minutes.

## Instructions:

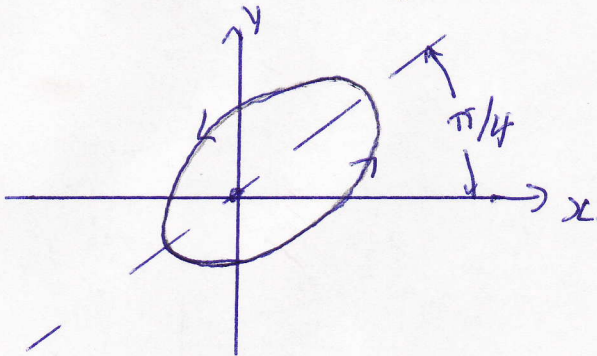
- Write your name and student ID on **every** page.
- This examination contains five questions worth a total of 40 points.
- Write each answer **very clearly** below the corresponding question (Use back of page if needed).
- No calculators, books, notebooks or any other written materials are allowed.
- **Good luck!**

1. (10 pts) Find the arclength of the curve parametrized by  $\mathbf{r}(t) = (a \cos t \sin t, a \sin^2 t, bt)$ ,  $t \in [0, 5]$  where  $a, b > 0$ .

2. (10pts) If a particle traverses the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  in the counterclockwise direction at constant speed 10, find the acceleration vector at the point  $(a, 0)$ .

3. (5 pts) State carefully: the definition of torsion at a point on a smooth curve.

4. (5 pts) A comet follows the elliptical path shown below. The sun is at the origin. Describe and draw a rough sketch of the path traced out by the comet's velocity vector, where the tail of the velocity vector is at the origin.



5. (10 pts) A particle moving in the  $x - y$  plane starts at  $t = 0$  from the point  $x = 1, y = 0$ . Let  $(r(t), \theta(t))$  denote its polar coordinates at time  $t$ . It is known that  $r(t) = (t + 1)^2$ , the speed of the particle at time  $t$  is  $v(t) = \sqrt{8}(t + 1)$ , and that  $\theta'(t) \geq 0$  for all  $t$ .

(a) Find  $\theta(t)$  for all  $t \geq 0$ .

(b) Find the first time  $t > 0$  when the particle lies on the  $x$  axis.