Practice Question

Let
\[ F(x, y, z) = \frac{m}{4\pi |r|^3}, \quad \text{for } r = (x, y, z) \neq (0, 0, 0). \]

Hence \( F \) may be viewed as a repulsive force from the origin obeying an inverse square law. But we will think of it as a mass flow v.f. for some "stuff".

(a) Show that \( \nabla \cdot F = 0 \).

(b) Let \( S_0 \) be a smooth oriented surface so that \( S_0 = \partial B \) for some solid \( B \) in \( \mathbb{R}^3 \), where \( \mathbf{0} \notin S_0 \) and \( S_0 \) is oriented by the outward normal from \( B \).
   (i) If \( \mathbf{0} \notin B \) show that \( \int \int_{S_0} F \cdot dS = 0 \).
   (ii) If \( \mathbf{0} \in B \) show that \( \int \int_{S_0} F \cdot dS = m \).

We therefore say \( F \) is point source of stuff at \( \mathbf{0} \) of intensity \( m \).

Hint for (ii): We essentially showed in class (March 23) that if \( S_a \) is the sphere \( x^2 + y^2 + z^2 = a^2 \), oriented by the outward normal, then \( \int \int_{S_a} F \cdot dS = m \). Now apply the Divergence Theorem to the part of \( B \) outside the ball of radius \( a \) for small enough \( a \) so that the latter is contained in \( B \).