Math 317, HW8, Due on Wednesday, March 28

(1) Let \( S \) be the hemisphere \( x^2 + y^2 + z^2 = a^2, z \geq 0 \). Let \( f(x,y,z) = z^2 \).

Find \( \int \int_S f dS \) by using the following parametrizations:

(a) (spherical coordinates)
\[
\mathbf{r}_1(\phi, \theta) = a(\sin(\phi) \cos(\theta), \sin(\phi) \sin(\theta), \cos(\phi)), \ 0 \leq \theta \leq 2\pi, \ 0 \leq \phi \leq \pi/2
\]

(b) (graph of a function)
\[
\mathbf{r}_2(x, y) = (x, y, \sqrt{a^2 - x^2 - y^2}), \ x^2 + y^2 \leq a^2
\]

(2) Section 17.7: 10, 14, 20, 26, 38, 45

For the last questions recall we discussed heat flow earlier in the course—see page 1126 in 17.7 for a refresher.

(3) Consider the following parametrization of (what turns out to be) the Möbius strip:
\[
\mathbf{r}(u, v) = 
\left(1 + \frac{1}{2}v \cos\left(\frac{u}{2}\right)\right) \cos(u), \ \left(1 + \frac{1}{2}v \cos\left(\frac{u}{2}\right)\right) \sin(u), \ \frac{1}{2}v \sin\left(\frac{u}{2}\right)
\]

defined on the domain \( D = \{(u, v): 0 \leq u \leq 2\pi, -1 \leq v \leq 1\} \).

(a) Show that the \( v \)-curves are straight line segments.
(b) Show that \( \mathbf{r}(0, v) = \mathbf{r}(2\pi, -v) \) for all \( v \).
(c) Show that the \( u \)-curve, corresponding to \( v_0 = 0 \) is a simple closed curve (in fact, a circle).
(d) Show that the \( u \)-curve, corresponding to \( v_0 = 1 \) is not a closed curve (i.e., has distinct endpoints).
(e) Show that the union of the two \( u \)-curves, corresponding to \( v_0 = \pm 1 \) is a simple closed curve.
(f) Compute \( \mathbf{r}_u, \mathbf{r}_v \) and \( \mathbf{r}_u \times \mathbf{r}_v \) at all points of \( D \)–here \( \mathbf{r}_u \) is the \( u \)-partial derivative of \( \mathbf{r} \).
(g) Show that the normal vector \( \mathbf{r}_u \times \mathbf{r}_v \) does not define a continuous vector field on the Möbius strip.