Math 317 Assignment 4

This assignment is due in class on Wed. Feb. 15. I will put relevant pages of the text on the webpage. It is suggested you do questions 1 and 2 before the midterm.

1. The moon orbits the earth in approximately a circular orbit a mean distance of 385,000 km from the centre of the earth. It takes about 27.3 days to complete an orbit. At what distance from the centre of the earth and in what plane should a satellite be inserted into circular orbit if it is to remain directly over the same position on earth at all times. (You can do this using values for G and the mass of the earth but the point here is to use Kepler's laws–which apply to objects in orbit about the earth as well–to derive the answer from only the given information).

2. Find the position vector $\mathbf{r}(t)$ of a particle that has acceleration vector $\mathbf{a}(t) = 2\mathbf{i} + 6t\mathbf{j} + 12t^2\mathbf{k}$, and initial position and velocity vectors $\mathbf{r}(0) = \mathbf{j}$ and $\mathbf{v}(0) = \mathbf{i}$, respectively. (Take a look at Example 3 on page 876.)

3. The density of a conical helix parameterized by $\mathbf{r}(t) = (t\cos t, t\sin t, t)$, $0 \le t \le 2\pi$, is $f(x, y, z) = 2z$. Find the mass of the conical helix.

4. A thin wire has the shape of the first quadrant part of the circle $x^2 + y^2 = a^2$. If the density function of the wire is $\rho(x, y) = 2xy$, find the mass and centre of mass of the wire.

5. Find the moment of inertia about the z-axis (that is, the value of $\int_C (x^2 + y^2)\rho(x, y)ds$) for a wire of constant density $\rho(x, y) = 3$ lying along the curve $\mathbf{r}(t) = (e^t \cos t, e^t \sin t, t)$, $0 \le t \le 2\pi$.

6. Find $\int_{\mathbf{C}} xdy + ydz + zdx$ along the oriented curve $\mathbf{C}$ of intersection between the planes $x - y + z = 0$ and $x + y + 2z = 0$ from the origin to the point $(3, 1, -2)$.

7. If $\mathbf{F}$ is a C^1 vector field in $\mathbf{R}^3$ show that at all points where $\mathbf{F} \neq \mathbf{0}$,

$$\frac{\partial}{\partial x}|\mathbf{F}| = \frac{\mathbf{F} \cdot \left(\frac{\partial}{\partial x}\mathbf{F}\right)}{|\mathbf{F}|}.$$

Here $\frac{\partial}{\partial x}\mathbf{F}$ is calculated componentwise.

8. Find the work done by a force $\mathbf{F}(x, y, z) = (y, z, -x)$ on a particle which traverses the first loop of the helical path parametrized by $\mathbf{r}(t) = (a\cos t, a\sin t, ct)$, $t \ge 0$.

9. p. 1081 # 42, 46 (see page 1071 for the latter)