Math 317 Assignment 2

This assignment is due in class on Wed. Jan.25.

1. Let C be the curve of intersection between $x^2 = ay$ and $z = \frac{2xy}{3a}$ where a is a positive constant. Find the length of the portion of the curve from the origin to $(a, a, \frac{2}{3}a)$.

2. Let C be parametrized by $\mathbf{r}(t) = (\sin t - t\cos t, \cos t + t\sin t, t^2)$, $t \geq 0$. Find the speed, the unit tangent vector, the principal unit normal vector and the curvature at $\mathbf{r}(t)$.

3. Find the curvature of $y = x^n$ at $(1, 1)$ and $(0, 0)$, where n is an integer, $n \geq 2$.

4. Find the curvature of the curves parametrized by the following functions at the given point:
   (a) $\mathbf{r}(t) = 3t^2\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}$, $t \in [0, 2\pi]$ at $\mathbf{r}(t)$.
   (b) $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t\mathbf{k}$, $t \geq 0$, at $(1, 1, 1)$.

5. If f is a C^2 function on an interval I find the principal unit normal vector for the curve $y = f(x)$ at $(x, f(x))$ in terms of $f'(x)$ and $f''(x)$ for all x where $f''(x) \neq 0$.

6. Find the normal plane and osculating plane for the curve parametrized by $\mathbf{r}(t) = (t, t^2, t^3)$ at $(1, 1, 1)$. **Hint:** In each case find a vector orthogonal to the given plane.

7. Let C be the curve parametrized by $\mathbf{r}(t) = (e^t \cos 2t, e^t \sin 2t, e^t)$, $t \geq 0$.
   (a) Find an arclength parametrization of C.
   (b) Find the curvature of C at $(1, 0, 1)$.
   (c) If a particle traverses C at constant speed 3 in the same direction as the above parametrization, find its acceleration vector as it passes through $(1, 0, 1)$.