2.

14.1.24. This is 11. The projection onto the x-y plane is the circle \( x^2 + y^2 = 1 \), so the curve is contained in the cylinder \( x^2 + y^2 = 1 \). \( \vec{v}(t) = \vec{v}_0 \) for \( \theta > 0 \) means the cars get closer as \( t \) increases.

25. \( x^2 + y^2 = (\cos t)^2 + (\sin t)^2 = t^2 \), and \( \sin^2 t \) is a cone \( x^2 + y^2 + z^2 = 1 \). The curve lies in the cone \( x^2 + y^2 + z^2 \). \( \dot{\vec{v}}(t) = \vec{v}_0 \) means the z coordinate increases with unit speed, producing an upward spiral in the above cone.

28. \( |\vec{v}(t)|^2 = 5 \Rightarrow \sin^2 t + \cos^2 t + t^2 = 5 \)

\[ 1 + t^2 = 5 \]
\[ t = \pm 2 \]

Not graded: \( \vec{r}(1-2) = (\sin 1, \cos 2, -2) = (1-\sin 2, \cos 2, -2) \)

and \( \vec{r}(1) = (\sin 2, \cos 3, 2) \) are the only 2 points of intersection.
36. \( x^2 + y^2 = 4 \Rightarrow 2 = xy \),

So this is the graph of the function \( z = \frac{2}{x+y} \), where \((x, y)\) is restricted to the circle of radius \( 2 \), \( x^2 + y^2 = 4 \).

\[ (x, y) = (4 \cos \theta, 2 \sin \theta) \quad \theta \in [0, 2\pi] \]

\[ z(t) = (4 \cos \theta \text{ for } \theta \in [0, 2\pi] \]

42. For collisions we need \( \bar{r}_1(t) = \bar{r}_2(t) \)

\( \Rightarrow t = \pm 2 \beta, \quad \beta^2 = 1 + 6t \) and \( \beta^3 = 1 + 14t \).

\( \Rightarrow t = -1 \), \( \beta^2 = 1 + 6t \) and \( \beta^3 = 1 + 14t \) which has no solutions.

There are no collisions.

For the paths to intersect we only require

\( \bar{r}_1(t) = \bar{r}_2(t) \) for some \( t \in \mathbb{R} \). (Assume the domain is as large as possible in absence of other information)

\( \Rightarrow t = \pm 2 \beta, \quad \beta^2 = 1 + 6t \) and \( \beta^3 = 1 + 14t \).

Path \( 1 \) and \( 2 \).

\( (x + 2\theta)^2 = 1 + 6t \Rightarrow 4 \theta + 4 \theta^2 = 6 t \Leftrightarrow 4 \theta^2 - 24 = 0 \)

\( \Leftrightarrow 24(2\theta - 1) = 0 \Rightarrow \theta = 0 \) or \( 4 = \frac{1}{2} \)

(1, 0) and \( (2, \frac{1}{2}) \). Both points satisfy (3).

The curves do intersect \( \bar{r}_1(t) = \bar{r}_2(t) \) at \( (1, 0) \) and \( (2, \frac{1}{2}) \).

14.2. \( \bar{r}(t) = (t^2 + 2t) + (\cos \theta \text{ for } \theta \in [0, 2\pi]) \)

\[ \bar{r}'(t) = 2t + \cos \theta \text{ for } \theta \in [0, 2\pi] \]

\( C \) is a circle center \((1, 2)\) and radius \( \sqrt{5} \).
\[ \mathbf{r}(t) = \langle \cos 3t, \sin 3t, t \rangle \]

\[ \mathbf{v}(t) = \langle -3\sin 3t, 3\cos 3t, 1 \rangle \]

14. \[ \mathbf{r}(t) = \langle a \cos 3t, b \sin 3t, c \rangle \]
\[ \mathbf{v}(t) = \langle -3a \sin 3t, 3b \cos 3t, 0 \rangle \]

To find the angle of intersection, we have
\[ \cos \theta = \frac{\mathbf{r}(1)}{\lVert \mathbf{r}(1) \rVert} \cdot \frac{\mathbf{v}(2)}{\lVert \mathbf{v}(2) \rVert} = \frac{-1 + 3}{\sqrt{6} \sqrt{18}} = \frac{2}{3} \]
\[ \theta = \arccos \left( \frac{2}{3} \right) \approx 54.7^\circ \text{ or } 0.96 \text{ radians} \]
4.7 \[ \frac{d}{dt} \left( f(t) \cdot g(t) \right) = f(t) \frac{d}{dt} g(t) + g(t) \frac{d}{dt} f(t) \] (product rule)

not graded

\[ \frac{d}{dt} \left( f(t) \cdot g(t) \right) = f(t) \frac{d}{dt} g(t) \]

4.8 \[ \frac{d}{dt} \left[ u(t) \cdot (v(t) \cdot w(t)) \right] = u(t) \cdot \frac{d}{dt} (v(t) \cdot w(t)) + \frac{d}{dt} u(t) \cdot (v(t) \cdot w(t)) \]

not graded

\[ = u(t) \cdot \left[ \frac{d}{dt} (v(t) \cdot w(t)) \right] + \frac{d}{dt} u(t) \cdot (v(t) \cdot w(t)) \]

= \[ u(t) \cdot \left[ \frac{d}{dt} (v(t) \cdot w(t)) \right] + \frac{d}{dt} u(t) \cdot (v(t) \cdot w(t)) \]

4.3 \[ y = \left( \frac{1}{\sin t}, \cos t, -\frac{1}{\cos t} \right) = \left(-\sin t, \cos t, -\cos t \right) \]

\[ \Rightarrow y'(t) = \left(1 + \tan^2 t \right)^{\frac{1}{2}} \]

3. \[ L = \int_{a}^{b} \sqrt{1 + \tan^2 t} \, dt = \int_{a}^{b} \sec t \, dt = \int_{a}^{b} \sec t \, dt \]

= \ln |\sec t + \tan t| \bigg|_{a}^{b} = \ln (|\sec t + \tan t|) - \ln (|\sec t + \tan t|)

= \ln (\sqrt{2} + 1) - \ln (1) = \ln (\sqrt{2} + 1) \]