This midterm has 6 questions on 8 pages, for a total of 40 points.

Duration: 75 minutes

- Read all the questions carefully before starting to work.
- You should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. None of the following are allowed: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

Full Name (including all middle names): __________________________________________

Student-No: ___________________________________________________________________

Signature: ____________________________________________________________________

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1. (a) Let \( g : X \to Y \) be a function. Define precisely what it means for \( g \) to be surjective.

(b) Give an example of a function \( h : [0, 2] \to [0, 1] \) which is injective but not surjective. Justify your example.

(c) Let \( f : E \to F \) be a function and let \( C \subseteq E \) and \( D \subseteq F \). Give precise definitions of the sets \( f(C) \) and \( f^{-1}(D) \).
2. Let $A, B, C$ be sets.

(a) Prove that if $A \subseteq B$ then $A - (B \cap C) \subseteq (A - C)$

(b) Disprove that $(A - B) - (B - C) = (A - B) - C$. 
5 marks  3. Use induction to prove that $3^n > n^3$ for every non-negative integer $n \geq 4$. 
4. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be defined by $f(x) = 1 - \frac{1}{x^2}$.

(a) Show that $f$ is not injective.

(b) Show that $f$ is not surjective.
(c) Now define

\[ g : \{ x \in \mathbb{R} | x > 0 \} \rightarrow \{ x \in \mathbb{R} \text{ s.t. } x < 1 \} \quad \quad \quad g(x) = 1 - \frac{1}{x^2} \]

Show that \( g \) is bijective.
5. Decide whether the following are true or false. Prove your answers

(a) If \( a_n = 1212 \ldots 12 \) (with \( n \) 12’s), then \( 12 | a_n \) for all \( n \in \mathbb{N} \).

(b) Let \( f : A \to B \) be a surjective function and let \( D \subseteq B \). Then

\[
D \subseteq f(f^{-1}(D))
\]
6. Prove that $[1, 2]$ is not well-ordered.