1. Let $A_1, A_2, B_1, B_2$ be non-empty sets such that $|A_i| = |B_i|$ for $i = 1, 2$. Prove that
   (a) $|A_1 \times A_2| = |B_1 \times B_2|$.
   (b) If $A_1 \cap A_2 = B_1 \cap B_2 = \emptyset$, then $|A_1 \cup A_2| = |B_1 \cup B_2|$.
   Remember — the sets may or may not be finite. This also applies to the remaining questions below.

2. Let $A$ be an non-empty set. Prove that $|A| \leq |A \times A|$.

3. (a) If $\mathcal{P}_{\text{fin}}(\mathbb{N})$ denotes the set of finite subsets of $\mathbb{N}$, show that $\mathcal{P}_{\text{fin}}(\mathbb{N})$ is denumerable.
   (b) If $\mathcal{P}_{\text{inf}}(\mathbb{N})$ denotes the set of infinite subsets of $\mathbb{N}$, show that $\mathcal{P}_{\text{inf}}(\mathbb{N})$ is uncountable.
   **Hint:** One of the questions from Workshop 5 may be helpful.

4. Let $A, B$ be sets. Prove that
   
   if $|A - B| = |B - A|$ then $|A| = |B|$.

   **Hint:** draw a careful picture

5. Let $\{0, 1\}^\mathbb{N}$ be the set of all possible sequences of 0’s and 1’s. Give a direct proof that $\{0, 1\}^\mathbb{N}$ is uncountable. (Do not just quote Theorem 10.14 to do this!)

6. Corollary 10.20 in the text states that if $\{0, 1\}^\mathbb{N}$ is the set of $\{0, 1\}$-valued sequences (or equivalently the set of function from $\mathbb{N}$ to $\{0, 1\}$), then $|\mathbb{R}| = |\{0, 1\}^\mathbb{N}|$. Use this to prove that $|\mathbb{R} \times \mathbb{R}| = |\mathbb{R}|$. 