• There are 9 questions worth a total of 32. Not all questions will be graded though.

1. 2 marks 2.46

2. 3 marks Let \( P \) be the statement \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ s.t. } y^2 = x' \).
   
   (a) State this in words (no notation allowed).
   (b) Is this statement true?
   (c) State the negation in symbols.

3. 6 marks 2.48 all parts except (d) and (h)

4. 2 marks 2.68 But change the definition of \( S \) to \( S = \{ (x, y) : x \in A, y \in B, P(x) \Rightarrow Q(y) \text{ is false} \} \).

5. 8 marks Let \( S = [1, 2] \) and \( T = (3, \infty) \). Determine the truth value of the following statements. Prove your answers—no proof = no marks
   
   (a) \( \exists x \in S \text{ s.t. } \exists y \in T \text{ s.t. } |x - y| > 3 \)
   (b) \( \exists x \in S \text{ s.t. } \forall y \in T, |x - y| > 3 \)
   (c) \( \forall x \in S, \exists y \in T \text{ s.t. } |x - y| > 3 \)
   (d) \( \forall x \in S, \forall y \in T, |x - y| > 3 \)

6. 3 marks Let \( I = \{ n^2 | n \in \mathbb{Z} \} \). Let \( P \) be the statement

   \[ \bigcup_{k \in I} [k, 2k] = \mathbb{R}. \]

   (a) Reformulate \( P \) using quantifiers.
   (b) Is \( P \) true or false? Prove or disprove.

7. 2 marks Let \( A = \{ x | \forall n \geq 3, 1/n < x < 1 - 1/n \} \).

   (a) Represent the set \( A \) as an intersection of an indexed collection of sets.
   (b) Find \( A \) (that is, represent it in the simplest possible form, using the notation for intervals in \( \mathbb{R} \)). No proof required.

8. 3 marks Suppose you have the following information about the population of the planet QE220:

   • Among the inhabitants of QE220 who can watch TV, not all have antennae on their head.
   • The inhabitants of QE220 that are green and do not have antennae, cannot watch TV.

   Does it follow that not all the inhabitants of QE314 that can watch TV are green? Justify your answer.
   
   **Hint.** Start by writing down the statement you are asked about.

9. 3 marks 3.6