Math 217 Practice Midterm 2

Instructor: Ed Perkins

Duration: 50 minutes.

Instructions:

• Write your name on every page, and student ID on page 1.

• This examination contains four questions worth a total of 50 points.

• Write each answer very clearly below the corresponding question (Use back of page if needed).

• No calculators, books, notebooks or any other written materials are allowed.

• Good luck!
1. (10 pts) Evaluate \( \iint_D (x^2 + y^2)^{3/2} \, dx \, dy \), where \( D \) is the region in the disk \( x^2 + y^2 \leq 1 \) above the graph of \( y = |x| \).
2. (13 pts)

(a) Define a critical point of a function \( f : \mathbb{R}^2 \to \mathbb{R} \).

(b) Find the critical points of \( f(x, y) = 2xye^{-x^2-y^2} \) and use the second derivative test to determine if they are local minima, local maxima or saddle points.
3. (13 pts)

(a) Define the gradient of a differentiable function $f(x, y)$ at the point $(1, 2)$.

(b) A mountain has the shape of an elliptic paraboloid $z = c - ax^2 - by^2$, where $a$, $b$ and $c$ are positive constants. An engineer building a railroad up the mountain wants to lay track at a 3% grade. In what direction(s) should the track be laid at the point $(1, 1)$. Make a sketch on the $x - y$ plane, showing the two possible directions.
4. (14 pts)

(a) Use the method of Lagrange multipliers to find the maximum area of a rectangle with opposite corners at (0, 0) and (x, y) where $x^4 + y^4 = 2$, assuming the maximum area exists.

(b) Prove in (a) that the maximum area does exist.