1. \( \mathbf{F}(t) = (acost, asint, ct) \quad t \in [0, 2\pi] \)

\[
\int_C x \, dy + y \, dz + z \, dx = \int_0^{2\pi} (acost) \, dt = \int_0^{2\pi} (acost) \, dt
\]

\[
= -a^2 \int_0^{2\pi} \sin^3 t \, dt + ac \int_0^{2\pi} \cos^3 t \, dt
\]

\[
= -a^2 \left[ \int_0^{2\pi} \sin t \, dt \right] + ac \left[ \int_0^{2\pi} \cos t \, dt \right]
\]

\[
= -a^2 \pi + 0 = -\pi a^2
\]

2. Line of intersection is \( z \) both normal vectors of the 2 planes

and so points in direction \( \hat{v} = \hat{u} \times \hat{n} = \begin{pmatrix} 1 & 3 & -1 \\ 1 & -1 & 1 \end{pmatrix} \)

\[
= \hat{i}(-2-1) - \hat{j}(2-1) + \hat{k}(1+1)
\]

\[
= (3, -1, 2) \quad \text{or} \quad (-3, 1, -2)
\]

Or this is obvious from given information that \( (3, 1, -2) \) lies on the line!

So \( \mathbf{F}(t) = (3t, t-2t) \) \( t \in [0, 1] \) parametrizes \( \mathbf{E} \).

\[
\int_C x \, dy + y \, dz + z \, dx = \int_0^1 (3t \cdot 1 + (t-2t) - 2t \cdot 3) \, dt
\]

\[
= \int_0^1 -5t \, dt = -\frac{5}{2}
\]
\( \text{(a)} \) Ellipsoid. Simply Connected.

\( \text{(b)} \) Complement of Ellipsoid. Simply Connected.

\( \text{(c)} \) Complement of infinite cylinder of radius 1 about \( z \)-axis.
Not simply connected.

\[ \mathcal{C}: \{ (x,y,z) \mid x^2 + y^2 = 1, \quad z \neq 0 \} \]

\( \mathcal{C} \) cannot be contracted to a point in \( \mathcal{D} \) in above region.

\( \text{(d)} \) \( \mathcal{D} = \{ (x,y,z) \mid x^2 + y^2 = 1 \} \) and \( z = 0 \) is circle of radius 1 in \( x-y \) plane centered about \( (0,0,0) \).

\( \mathcal{D} \) is complement of above circle.
Not simply connected.

\[ \mathcal{C}: (y-1)^2 + z^2 = 1, \quad z = 0. \]

\( \mathcal{C} \) cannot be contracted to a point in \( \mathcal{D} \).

\( \text{(e)} \)

\[ x^2 + y^2 = 1 \]

(\( a \)) \( \frac{\partial P_x}{\partial x} = e^x \cos y + 2 \quad \frac{\partial P_y}{\partial y} = e^x \cos y + 2. \]

\( \therefore P \) is conservative.
(b) Let \( G(x, y) = F(x, y) + (y, 0) \) as in (4).

\[
\int \overline{E} \cdot d\mathbf{n} = \int \overline{E} \cdot d\mathbf{n} + \int y \, dx = \int y \, dx \quad \text{by (6)}
\]

\[
= \int_{E} y \, dx + \text{area } \left( \text{Il } \overline{E} - \overline{C} \right)
\]

\[
= -\text{Area } \left( \text{Il } \overline{E} - \overline{C} \right)
\]

\[
= -\pi \times \frac{1}{2} = -\pi
\]

S. p995 #24 Let's directly find a \( \alpha u \) of \( \mathbf{F} = \mathbf{0} \).

Wind \( \alpha = \frac{2}{\rho^2} \) : \( \alpha = \frac{2}{\rho^2} \sum \frac{Z}{x^2 + y^2 + z^2} \, dx = \frac{1}{2} \int \frac{1}{x^2 + y^2 + z^2} \, dV = \frac{4\pi}{2} \)

\[
= \frac{1}{2} \ln(1 + y^2 + z^2) = \frac{1}{2} \ln \left( 1 + \left( \frac{1}{2} \right)^2 \right) \quad \text{on } \mathbb{R}^3 \setminus \{(0,0)\}
\]

We have your thee addicive constants as clearly

\[
\alpha = \frac{2}{x^2 + y^2 + z^2}, \quad \alpha = \frac{2}{x^2 + y^2 + z^2}, \quad \alpha = \frac{2}{x^2 + y^2 + z^2}
\]

ie. \( \mathbf{F} \) is conservative on \( \mathbb{R}^3 \setminus \{(0,0)\} \) on any \( \mathbf{D} \) in \( \mathbb{R}^3 \setminus \{(0,0)\} \).
\[ \text{Q. 14.4 # 29.} \]
\[
\oint_{C} (2x + 2y^2) \, dy - (4y^2 + e^{2x}) \, dx
\]
\[
= \int_{0}^{1} \int_{0}^{1} \left( \frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial x} \right) \, dA = \int_{0}^{1} \int_{0}^{1} 2 + 8y \, dy \, dx
\]
\[
= 2 + 8 \int_{0}^{1} y \, dy = 2 + 4 = 6.
\]
\[ \text{# 30} \]
\[
\oint_{C} (2x - 3y) \, dy - (3x + 4y) \, dx
\]
\[
= \int_{0}^{1} \int_{0}^{1} \left( \frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial x} \right) \, dA = \int_{0}^{1} \int_{0}^{1} 2 + 7 \, dy \, dx = 6
\]
\[
= \int_{0}^{2\pi} \int_{0}^{1} (2 + 4) r \, dr \, d\theta = 12\pi \int_{0}^{1} r \, dr = 6\pi
\]