Math 217 Assignment 4

This assignment is due in class on Oct. 11. You should make sure you can do them before the midterm on Oct. 4.

1. Find \( \text{Int}(D) \), \( \partial D \) and \( \overline{D} \) for the following subsets of the plane (you need not justify your answers):
   (a) \( D = \{ (x, y) \in \mathbb{R}^2 : xy > 0 \} \)
   (b) \( D = \{ (x, 0) \in \mathbb{R}^2 : 1 \leq x < 2 \} \).

Do the same for the following subsets of \( \mathbb{R}^3 \):
   (c) \( D = \{ (x, y, z) \in \mathbb{R}^3 : 0 < x^2 + y^2 + z^2 < 4 \} \)
   (d) \( D = \{ (x, y, z) \in \mathbb{R}^3 : z = 0, x^2 + y^2 < 4 \} \).

2. Sec. 12.3 #20, 28

3. Sec. 12.3 Consider the function in #34. Is it possible to define \( S(0,0) \) so that it is continuous at \( (0,0) \)? Explain your answer.

4. Sec. 12.4 #28

5. Sec. 12.7 #18, 24

6. Let

\[
f(x, y) = \begin{cases} 
\frac{2xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x, y) \neq (0,0); \\
0, & \text{if } (x, y) = (0,0).
\end{cases}
\]

(a) Find the first order partial derivatives of \( f \), first for \( (x, y) \neq (0,0) \) and then \( (x, y) = (0,0) \).

(b) Find \( f_{xy}(0,0) \) and \( f_{yx}(0,0) \) and note they are NOT equal. This does not contradict the equality of mixed partials proved in class since \( f_{xy} \) is not continuous at \( (0,0) \).

Additional Practice Problems (Not to hand in):

12.3 #8, 52, 36
12.4 #36
12.7 #16, 30

Find \( \text{Int}(D) \), \( \partial D \) and \( \overline{D} \) if \( D = \{ (x, y, z) \in \mathbb{R}^3 : z = x^2 + y^2, x^2 + y^2 \leq 1 \} \). What if \( D = \{ (x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq x^2 + y^2, x^2 + y^2 \leq 1 \} \)?