Math 217 Assignment 11

This assignment is NOT to be handed in but make sure you can do the questions. Solutions to most of the problems will be posted on the web.

1. Let $A$ be the area of a smooth surface $D$ forming a part of a sphere of radius $R$ centered at the origin. Let $V$ be the volume of the solid “cone” consisting of all lines joining the origin to $D$. Show that $V = (1/3)AR$.

2. Let $B$ be a bounded solid in $\mathbb{R}^3$ such that $\partial B$ is a smooth closed surface oriented by the outward normal. If $\vec{F}$ is a $C^2$ v.f. on an open set containing $B \cup \partial B$ show that the flux of $\vec{\nabla} \times \vec{F}$ through $\partial B$ is zero:
   (a) by the Divergence Theorem.
   (b) by Stokes’ Theorem.

3. 14.8 #14, 26

4. 14.7 #8, 12, 41

5. (a) 14.5 #71
   (b) 14.8 #28, 39(a), (b)

6. Let 
   $$\vec{F}(x, y, z) = (e^{x^5 \sin(x)} - y^2 - z^2, xy + z, xz + y)$$

Compute $\int_{\vec{C}} \vec{F} \cdot d\vec{r}$ where $\vec{C}$ is the curve parametrized by:

   $$\vec{r}(t) = \begin{cases} 
   (\sin(t), 0, \cos(t)) & \text{if } 0 \leq t \leq \pi/4 \\
   \left( \frac{\cos(t-\pi/4)}{\sqrt{2}}, \frac{\sin(t-\pi/4)}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) & \text{if } \pi/4 \leq t \leq 3\pi/4.
   \end{cases}$$

Hint: Complete $\vec{C}$ to a simple closed curve which bounds a nice surface.