**Theorem 5.6.** Suppose $A$ and $B$ are two nonempty sets of reals such that $a \leq b$ for all $a \in A$ and $b \in B$.

(a) $A$ is bounded above and hence has a least upper bound. $B$ is bounded below and hence has a greatest lower bound.

(b) $\sup A \leq \inf B$.

Proof. (a) Any $b$ in $B$ is an upper bound for $A$ and any $a \in A$ is lower bound for $B$. So the Completeness Axiom shows $\sup A$ exists, and Theorem 5.4 shows $\inf B$ exists.

(b) For any $b \in B$, $b$ is an upper bound for $A$ and so the least upper bound of $A$ is less than or equal to $b$. This shows that $\sup A$ is a lower bound for $B$. Hence $\sup A$ is less than or equal to the greatest lower bound of $B$.

$\square$