Practice Midterm 1      February 4, 2020      Duration: 50 minutes

This test has 4 questions on 8 pages, for a total of 50 points.

- Write your name or student number on every page.
- Attempt to answer all questions for partial credit.
- You may use the back of the previous page, or the blank page at the end, if you need more room.
- You may use one standard size page of notes (both sides) for reference purposes. No other aids of any kind are allowed, including electronic devices (such as calculators, phones, etc.)

First Name: ___________________________ Last Name: ___________________________

Student No.: ____________________________

Signature: ____________________________

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>18</td>
<td>8</td>
<td>8</td>
<td>16</td>
<td>50</td>
</tr>
<tr>
<td>Score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBC card for identification.

2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.

4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

5. Examination candidates suspected of any of the following; or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

(i) speaking or communicating with other examination candidates, unless otherwise authorized;

(ii) purposely exposing written papers to the view of other examination candidates or imaging devices;

(iii) purposely viewing the written papers of other examination candidates;

(iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

(v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).
1. Evaluate the following:

(a) \( \int_{0}^{\pi/4} (\sin 2x)^5 \cos 2x \, dx \).

(b) \( \int \frac{x+1}{x^2-x} \, dx \) (for \( x \neq 0 \) or 1).
6 marks

(c) \( \int_{1}^{e} x (\log x) \, dx. \)
2. True or False. If True provide a short proof using any results from the course or assignments. If False, provide, and justify, a counter-example.

(a) Any continuous function $f : [a, b] \rightarrow [0, \infty)$ has an antiderivative.

(b) Let $[x]$ denote the greatest integer less than or equal to $x$. Then $f(x) = ([x] - [x^2]) + e^{\sin x}$ is integrable on $[0, 8]$. 4 marks
3. Evaluate the following limit (and show it exists). Your answer should be an explicit real number.

\[ \lim_{n \to \infty} \sum_{j=1}^{n} \frac{j}{n^2} \sqrt{1 + \frac{j^2}{n^2}}. \]
4. (a) Carefully state the Fundamental Theorem of Calculus (both parts).

5 marks (b) Let $f$ be a differentiable function on the real line such that $f(0) = 1$ and $f'(0) = 2$. If $F(x) = \int_{-f(x)}^{f(x)} e^{t^2} \, dt$, find $F'(0)$. Justify your answer.
(c) Let $f, g : [0, 1] \to \mathbb{R}$ be continuous functions satisfying $\int_{x_1}^{x_2} f(u) \, du \leq \int_{x_1}^{x_2} g(u) \, du$ for all $0 \leq x_1 < x_2 \leq 1$. Prove that $f(x) \leq g(x)$ for all $x \in [0, 1]$. 

5 marks
This page has been left blank for your rough work and calculations.