Math 121 Assignment 9 Due Wed. March 25 at start of class—Note Date

You should do questions 1-4, and the practice questions in preparation for the midterm on March 18.
You may use any results stated in class (except when asked to prove such a result!).
This assignment has 2 pages.

1. Determine whether or not the following series converge or diverge to \( +\infty \).
   Justify your answers using the appropriate convergence tests:
   \[
   \begin{align*}
   (a) & \sum_{n=1}^{\infty} \frac{n+n^{1/3}}{n+n^{3/5}}. \\
   (b) & \sum_{n=1}^{\infty} \frac{(2n)!6^n}{(3n)!}. \\
   (c) & \sum_{n=1}^{\infty} \frac{1+(n!)}{(1+n)!}. \\
   (d) & \sum_{n=2}^{\infty} \frac{1}{(\log n)^n}.
   \end{align*}
   \]

2. Use the integral test error bounds to find \( N \) so that the error in approximating
   \( \sum_{n=1}^{\infty} \frac{1}{n} - \frac{4}{n} \) by \( \sum_{n=1}^{N} \frac{1}{n} - \frac{4}{n} \) is at most \( \frac{1}{1000} \). You should justify your answer and of course try to make \( N \) as small as possible.

3. The following question was left open in the lectures:
   Prove that \( \sum_{n=0}^{\infty} \frac{e^n n!}{n^n} = \infty \).
   **Hint:** First prove that \( \sum_{k=1}^{n} \log k \geq \int_{1}^{n} \log x \, dx \) for all natural numbers \( n \).
   Now use this to show the terms being summed in the above series are all bounded below by \( e \).

4. (a) If \( \lim_{n \to \infty} a_n = L \), prove that
   \[
   \lim_{n \to \infty} \frac{a_1 + a_2 + \ldots + a_n}{n} = L.
   \]
   (b) Give an example showing that the converse to (a) does not hold in general.

5. Determine whether the following series converge absolutely, converge conditionally or diverge. As always you need to justify your answers.
   \[
   \begin{align*}
   (a) & \sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{\log(n+1)}}. \\
   (b) & \sum_{n=1}^{\infty} \frac{(-2)^n}{(n!)^2}.
   \end{align*}
   \]

6. Find the smallest integer \( n \) for which the \( n \)th partial sum approximates the series sum \( s = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \) with an error of no more than \( .001 \).

7. Determine the centre, interval and radius of convergence of the following power series:
8. Find the sum of the given series or show that the series diverges:

(a) \( \sum_{n=1}^{\infty} \frac{e^n}{n^3}(1-2x)^n \).

(b) \( \sum_{n=0}^{\infty} \frac{x^{3n}}{\sqrt{n+1}} \). **Hint.** Try setting \( y = x^3 \).

9. Practice (not to hand in): Ch. 9. Sec. 9.3 #14 #42