Math 121 Assignment 11

This assignment is not to be handed in. Solutions will be posted.

1. Ex. 9.7 # 7 (i.e., Use Taylor series to estimate \( \cos(5^\circ) \) to within \( 5 \times 10^{-5} \).)

2. Transform the polar equation into Cartesian co-ordinates and identify the curve:

\[ r^2 = \csc 2\theta. \]

3. Sketch the following polar graph: \( r = 1 + 2 \cos \theta \) (This is Ex 8.5 #15.)

4. Find the area of the region inside the cardioid \( r = 1 + \cos \theta \) and outside the circle \( r = 3 \cos \theta. \) (Ans. \( \pi/4 \).)

5. Show the total arc length of the lemniscate \( r^2 = \cos 2\theta \) is \( 4 \int_{\pi/4}^{\pi/4} \sqrt{\sec 2\theta} \, d\theta. \)

6. Find the centroid and volume of the cone: \( x^2 + y^2 \leq z^2, 0 \leq z \leq a, \) where \( a > 0 \) is a constant. Ans. \((0,0,\frac{3}{4}a)\); \( V = \pi a^3/3. \)

7. Find the mass and centre of mass of the region \( 0 \leq y \leq 4 - x^2 \) if the density at \((x,y)\) is \( 2y. \)

8. Consider the planar region \( R \) illustrated in Q16(c) in Sec. 7.5 in the text but with \( x \geq 0. \)

   (a) Use Pappus’s theorem and Q6 above to find the x-coordinate of the centroid of \( R. \)

   (b) Consider the lower triangular portion of \( R, \) call it \( T. \) Use the formula for the centroid of a triangle on p. 420 of the text to find the centroid of \( T. \) Now use this and the centroid of the quarter circle (derived in class) to find the centroid of \( R. \)

9. Find the centroid of the region \( 0 \leq y \leq \sin x, 0 \leq x \leq \pi. \)

10. A sequence \( \{a_n\} \) is called Cauchy iff \( \forall \varepsilon > 0 \) there is an \( N \) so that for \( m,n > N, \)

    \[ |a_n - a_m| < \varepsilon. \]

    (a) Prove that any convergent sequence is Cauchy.

    (b) Prove that any Cauchy sequence is bounded.

The converse to (a) is true. In fact it is equivalent to the completeness axiom of the reals. But you don’t have to prove this. (One proof would use the existence of \( \limsup a_n \) for any bounded sequence—in particular for a Cauchy sequence.)