Math 121 Assignment 10 Due Wed. April 3 at start of class

You should only hand in Questions 1-4. All questions should be done as you prepare for the final exam. You may use any results stated in class (except when asked to prove such a result!).

1. Find the area of the region inside the cardioid \( r = 1 + \cos \theta \) and outside the circle \( r = 3 \cos \theta \).

2. Express the total arc length of the lemniscate \( r^2 = \cos 2\theta \) in terms of a definite integral. You need not evaluate the integral but make sure to simplify the integrand as much as possible.

3. Find the mass and centre of mass of the region \( 0 \leq y \leq 4 - x^2 \) if the density at \((x, y)\) is \( 2y \).

4. Find the centroid and volume of the cone: \( x^2 + z^2 \leq y^2, 0 \leq y \leq a \), where \( a > 0 \) is a constant.

END OF PART TO BE HANDED IN.

5. Find all \( x \) for which the following power series converges and evaluate the series for each such \( x \): \( \sum_{k=0}^{\infty} \frac{(-1)^k x^{3+6k}}{(1+2k)^{2k}} \).

6. Transform the polar equation into Cartesian co-ordinates and identify the curve:

\[ r^2 = \csc 2\theta. \]

7. Sketch the following polar graph: \( r = 1 + 2 \cos \theta \).

8. Find the centroid of the region \( 0 \leq y \leq \sin x, 0 \leq x \leq \pi \).

9. Use Taylor series to estimate \( \cos(5^\circ) \) to within \( 5 \times 10^{-5} \).

10. Consider the planar region \( R \) illustrated in Q16(c) in Sec. 7.5 in the text but with \( x \geq 0 \).

   (a) Use Pappus’s theorem and Q4 above to find the x-coordinate of the centroid of \( R \).

   (b) Consider the lower triangular portion of \( R \), call it \( T \). Use the formula for the centroid of a triangle on p. 420 of the text to find the centroid of \( T \). Now use this and the centroid of the quarter circle (derived in class) to find the centroid of \( R \).

11. Practice: Sec. 7.5 # 15