Math 121 Assignment 1 Due Wed. Jan. 9 at start of class

1. Evaluate (show work as always): \( \sum_{i=2}^{n} \ln \left( 1 + \frac{1}{i} \right) \) for \( n \geq 2, n \in \mathbb{N} \).

2. For each of the following sets find (a) the sets \( \mathcal{L} \) and \( \mathcal{U} \) of lower bounds and upper bounds, respectively, (b) the set’s least upper bound and greatest lower bound, if they exist. You need not provide justifications.

   (a) \( A = \{ \frac{1}{n} : n \in \mathbb{N} \} \)

   (b) \( B = \{ x \in \mathbb{R} : x \text{ rational}, 0 < x^2 \leq 2 \} \)

   (c) \( C = \{ \frac{1}{n} + (-1)^n : n \in \mathbb{N} \} \).

The following questions require rigorous proofs.

3. Suppose \( A \) and \( B \) are two nonempty sets of reals such that \( a \leq b \) for all \( a \in A \) and \( b \in B \).

   (a) Prove that \( A \) is bounded above and hence has a least upper bound. Prove that \( B \) is bounded below and hence has a greatest lower bound.

   (b) Prove that \( \sup A \leq b \) for all \( b \in B \).

   (c) Prove that \( \sup A \leq \inf B \).

4. Let \( A \) be a non-empty set bounded above and let \( u = \sup A \). Prove that for every \( \epsilon > 0 \), the set \((u - \epsilon, u] \cap A\) is non-empty. [This is easy to prove from the definition but is a useful result to have at your fingertips.]

5. These are practice questions NOT TO BE HANDED IN.

   Ex. 5.1 # 12, 22, 39