The total was scored out of 15 (Q1=3, Q2=2, Q4=2, Q5=3, Q6=2, Q7=3) and the average was 10.5/15. There was one 15/15 and one 14.5/15, so good job to the two of you. Q3 was just for “participation”...I found it very challenging to give/take away points for this question. I will say that I was fairly disappointed overall with the quality of work on this HW. You had 3 weeks to work on it, so I had fairly high expectations. I suppose the marks reflected that. (More than) a few notes:

• A very common issue that I noticed throughout this homework was the treatment of limits. Firstly, in order to use the sum rule for limits, you need to first verify that both limits exist! The same is true for when you are using linearity of the integral to break up an improper integral! Another major issue that I noticed is taking care to not mix up the variables of limits. For instance

\[
\int_{-\infty}^{\infty} f = \lim_{a,b \to \infty} \int_{-a}^{b} f = \lim_{a \to \infty} \int_{-a}^{0} f + \lim_{b \to \infty} \int_{0}^{b} f \neq \lim_{a \to \infty} \left[ \int_{-a}^{0} f + \int_{0}^{a} f \right].
\]

You can see why you might run into problems if you do this say if \( f = \frac{x}{1+x^2} \) (the Cauchy distribution). \( f \) is indeed an odd function, but nevertheless, the integral \( \int_{-\infty}^{\infty} f \) doesn’t exist which I believe you have seen in class. This same rule also holds for when you use IBP to solve an improper integral. The limit of integration is the same on all parts of the integral.

• I do expect you to prove (at least explain) claims like \( e^{-x^4} \leq e^{-x} \). This can be done very quickly by noting that the function in question is increasing. But I do expect some sort of proof.

• For 1a, why would you want to split the integral up? \( e^{-x^4} \) is continuous with no singularity. It is just a proper integral!

• There were quite a few marks lost on 1(b). I would encourage everyone to read the solutions for this one.

• For 2, It is very important that you use left/right hand limits as log is only defined on \((0, \infty)\). Please never write \( \log(0) \). This holds no meaning. Instead you should always work with limits. Note that the function is indeed odd, but that doesn’t mean you can say the integral is 0 (why?)

• For 4, you need to remark on the fact that \( f'' \) is continuous to use IBP and use FTOC to say \( \int f'' = f' \).

• For 5a, you again need to prove claims of inequalities of functions. You can use the fact proved in Math 120 that

\[
\lim_{t \to \infty} t^n e^{-t} = 0 \quad \text{for } n \in \mathbb{N}.
\]

Then use this say for \( n = \lfloor x \rfloor + 2 \) for instance. But even with this, in order to use comparison test, you need to break up the integral into THREE pieces: \([0, 1], [1, R], [R, \infty)\) where \( R \) is chosen so that for all \( t \geq R, f_x(t) < 1/t^2 \) say. You could do the question other ways as well, but they still need to be justified carefully.
• I would say that you guys over-thought Q6. Here’s a (brief) proof that I like: Let \( \epsilon > 0, M > 0 \) such that \( |f(x)| < M \) for all \( x \in [a,b] \) (since \( f \) is bounded). Let \((a, b) \ni c < \epsilon/(4M) + a\). Let \( P \) be a partition of \([c, b]\) such that \( U(f, P) - L(f, P) < \epsilon/2 \), and let \( P' = P \cup \{a\} \). Then
\[
U(f, P') - L(f, P') = (M_1 - m_1)(c - a) + U(f, P) - L(f, P) < 2M(c - a) + \epsilon/2 < \epsilon.
\]
This proves the claim and is also very concise. I would encourage you all to try to write up a proof of this problem as concisely as possible.

• For 7b, you should just use a previous homework problem. For 7c, you should use the comparison test for one of the directions and linearity for the other 2. For 7d, you really need to use part (c). That is what it is there for. 7(e) was worth at most 1 point, but I was very picky here.

Sorry about the essay, but there were lots of important issues to point out. Don’t worry too much if you didn’t do as well as you’d hoped. Try your best in the next one to make it up!