1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

\[ u_t = u_{xx} + e^{-4t} \sin(x) + 1, \quad 0 < x < \frac{\pi}{2}, \quad t > 0 \]

\[ u(0, t) = t, \quad u_x \left( \frac{\pi}{2}, t \right) = 1 \]

\[ u(x, 0) = x \]

by using an expansion in terms of the appropriate eigenfunctions. \[50 \text{ marks}\]

2. Consider the following initial boundary value problem for the damped wave equation with damping coefficient \(0 < \gamma < 1\):

\[ u_{tt} + 2\gamma u_t = u_{xx}, \quad 0 < x < \frac{\pi}{2}, \quad t > 0 \]

\[ u_x (0, t) = 1, \quad u_x \left( \frac{\pi}{2}, t \right) = \frac{\pi}{2} \]

\[ u(x, 0) = x, \quad u_t (x, 0) = \cos (5x) \]

a) Determine the steady state solution \(w(x)\).

b) Let \(u(x, t) = w(x) + v(x, t)\) and determine the corresponding boundary value problem for \(v(x, t)\).

c) Use the method of separation of variables to solve for \(v(x, t)\) and therefore \(u(x, t)\). \[50 \text{ marks}\]