1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

\[ u_t = u_{xx} + 1, \quad 0 < x < \frac{\pi}{2}, \quad t > 0 \]
\[ u(0, t) = t, \quad u_x\left(\frac{\pi}{2}, t\right) = e^{-\gamma t}, \quad 0 < \gamma < 1 \]
\[ u(x, 0) = x \]

by using an appropriate expansion in terms of the appropriate eigenfunctions. \[60 \text{ marks}\]

2. Consider the following initial boundary value problem for the wave equation:

\[ u_{tt} = u_{xx} + \gamma \sin(x) + x/\pi, \quad 0 < x < \pi, \quad t > 0 \]
\[ u(0, t) = 0, \quad u(\pi, t) = \frac{t^2}{2} \]
\[ u(x, 0) = 0, \quad u_t(x, 0) = \sin(3x) \]

a) Determine a simple function \( w(x, t) \) that satisfies the inhomogeneous boundary conditions.
b) Let \( u(x, t) = w(x, t) + v(x, t) \) and determine the corresponding boundary value problem for \( v(x, t) \).
c) Use an eigenfunction expansion (or extract a steady state solution for the boundary value problem for \( v(x, t) \) and use separation of variables) to solve for \( v(x, t) \) and therefore \( u(x, t) \). \[40 \text{ marks}\]

d) **Bonus Marks:** Assuming \( \gamma = 0 \), use D’Alembert’s solution (see the formula sheet) to determine the corresponding \( v(x, t) \) and therefore \( u(x, t) \). \[5 \text{ marks}\]