1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

\[ u_t = u_{xx} + x, \quad 0 < x < \pi/2, \quad t > 0 \]
\[ u(0, t) = e^{-\gamma t}, \quad u_x(\pi/2, t) = t, \quad \text{where } \gamma > 1 \]
\[ u(x, 0) = 1 + \sin 3x \]

by using an appropriate expansion in terms of the appropriate eigenfunctions.

\[ 60 \text{ marks} \]

2. Consider the following initial boundary value problem for the wave equation:

\[ u_{tt} = u_{xx} - 2, \quad 0 < x < 1, \quad t > 0 \]
\[ u(0, t) = 0, \quad u(1, t) = 0 \]
\[ u(x, 0) = x^2 - x, \quad u_t(x, 0) = \sin 2\pi x \]

a) Determine the steady state solution \( w(x) \).

b) Let \( u(x, t) = w(x) + v(x, t) \) and determine the corresponding boundary value problem for \( v(x, t) \).

c) Use the method of separation of variables to solve for \( v(x, t) \) and therefore \( u(x, t) \).

d) Now use D’Alembert’s solution (see the formula sheet) to determine \( v(x, t) \) and therefore \( u(x, t) \).

\[ 40 \text{ marks} \]