1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

\[ u_t = u_{xx} - u + e^{-t} \cos(x), \quad 0 < x < \frac{\pi}{2}, \quad t > 0 \]

\[ u_x(0, t) = e^{-t}, \quad u(\frac{\pi}{2}, t) = 0 \]

\[ u(x, 0) = x \]

by using an appropriate expansion in terms of the appropriate eigenfunctions.

[50 marks]

2. Consider the following initial boundary value problem for the damped wave equation with damping coefficient \(0 < \gamma < 1\):

\[ u_{tt} + 2\gamma u_t = u_{xx}, \quad 0 < x < \pi, \quad t > 0 \]

\[ u(0, t) = 0, \quad u(\pi, t) = 1 \]

\[ u(x, 0) = x/\pi, \quad u_t(x, 0) = \sin(x) \]

a) Determine the steady state solution \(w(x)\).

b) Let \(u(x, t) = w(x) + v(x, t)\) and determine the corresponding boundary value problem for \(v(x, t)\).

c) Use the method of separation of variables to solve for \(v(x, t)\) and therefore \(u(x, t)\).

[50 marks]