Math 257/316, Midterm 1, Section 102
4 pm on 8 th October 2014

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided.

Maximum score 50.

1. Consider the second order differential equation:

\[ Ly = 6x^2 y'' + 7xy' + (x - 1)y = 0 \]  \hspace{1cm} (1)

(a) Classify the points \(0 < x < \infty\) as ordinary points, regular singular points, or irregular singular points. For any regular singular points determine the roots of the corresponding indicial equation.

\[ \text{[7 marks]} \]

(b) If you were given \(y(1) = 1\) and \(y'(1) = 0\), what form of series expansion would you assume (Do not determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series?

\[ \text{[3 marks]} \]

(c) Use the appropriate series expansion about the point \(x = 0\) to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case.

\[ \text{[20 marks]} \]

2. Apply the method of separation of variables to determine the solution to the one dimensional heat equation with the following periodic boundary conditions:

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad -\pi < x < \pi, \ t > 0 \]

BC : \( u(-\pi, t) = u(\pi, t) \) and \( \frac{\partial u(-\pi, t)}{\partial x} = \frac{\partial u(\pi, t)}{\partial x} \)

IC : \( u(x, 0) = x \)

\[ \text{[20 marks]} \]